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CCXXXIII.

(Vol. XI.—February and March, 1882.)

DISCUSSIONS

UPON

WROUGHT IRON COLUMNS, TESTS AND FORMULÆ.

NOTE. [See Paper CCXXXII immediately preceding, EXPERIMENTS UPON PHŒNIX COLUMNS: by CLARKE, REEVES & Co. See also Paper CCXII, Vol. IX, p. 447. THE STRENGTH OF WROUGHT IRON COLUMNS: by G. BOUSCAREN.]

DISCUSSIONS BY G. BOUSCAREN, THEODORE COOPER, D. J. WHITTEMORE, CHARLES E. EMERY, DE VOLSON WOOD, C. L. STROBEL, A. S. C. WURTELE, WILLIAM H. BURR, MANSFIELD MERRIMAN, C. L. GATES, JAMES E. HOWARD and THOMAS C. CLARKE.

Discussion by G. Bouscaren, M. A. S. C. E.

The strength per square inch of a wrought-iron post depends principally on the three elements :

1. Ratio of length to diameter.
2. Shape of the cross-section.
3. Quality of the metal.

To these may be added as incidental causes of variation :

4. Workmanship, where the post is made of several parts riveted or bolted together.
5. Conditions in which the load is applied.

A formula expressing corretly the law governing the resistance of posts must necessarily contain representative factors of elements 1, 2 and 3.

In the empirical formula deduced from Hodgkinson's experiments by Gordon $\frac{P}{S} = \frac{f}{1 + a\left(\frac{l^2}{d^2}\right)}$, f and a are coëfficients dependent respectively upon the ultimate resistance to crushing and on the modulus of elasticity of the metal; the shape of the cross-section is left out altogether. The correctness of the formula must, therefore, be limited to solid posts of circular and square shapes of cross section, which were the types used by Hodgkinson in his experiments, the constants f and a being determined experimentally for every different kind of iron.

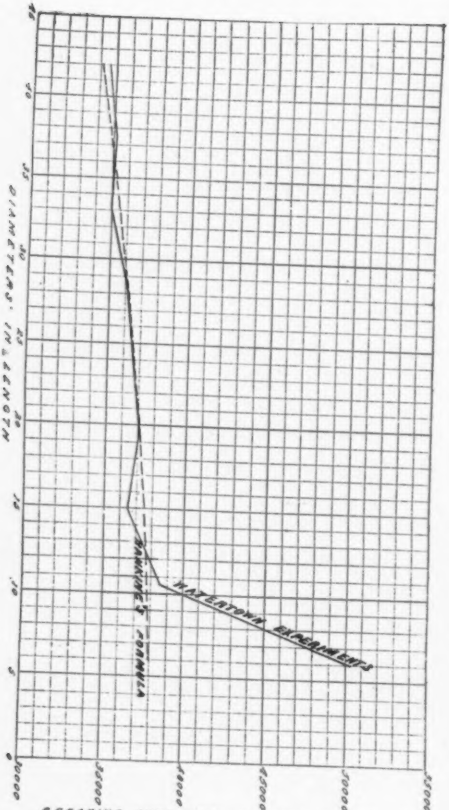
In Gordon's formula, modified as proposed by Rankine $\frac{P}{S} = \frac{f}{1 + a\frac{l^2}{r^2}}$.

The shape of the cross-section is represented by the radius of gyration r and the formula can be made to agree very approximately with the results of experiments by a proper selection of the constants. This was

TABLE SHOWING THE RESULTS OF RANKINE'S FORMULA AS APPLIED TO PHŒNIX POSTS TESTED AT WATERTOWN ARSENAL, NOVEMBER, 1879.

1	2	3	4	5	6	7	8	9	10
No. of Experiment.	Length of Posts.	Ratio of Length to Diameter.	Square of the Radius of Gyration.	$\frac{P}{S}$ by the Experiments.	Difference between Actual Resistances of Similar Posts.	Values of f. $f = \frac{P}{S}\left(1 + \frac{l^2}{36\,000\,r^2}\right)$	Values of f. $f = \frac{P}{S}\left(1 + \frac{l^2}{100\,000\,r^2}\right)$	$\frac{P}{S} = \frac{38\,000}{1 + \frac{l^2}{100\,000\,r^2}}$	Difference between Actual and Calculated Values of $\frac{"P"}{S.}$ (Columns 5 & 9.)
1	28 feet.	42	9 945	35 150	1 000	46 234	39 139	34 130	— 1 020
2	28 "	42	9 952	34 150		44 911	38 024	34 130	— 20
3	25 "	37½	9 953	35 270	230	44 129	38 459	34 850	— 420
4	25 "	37½	9 947	35 040		43 847	38 210	34 850	— 119
5	22 "	33	9 960	35 570	1 210	42 485	38 059	35 510	— 60
6	22 "	33	9 957	34 360		41 041	36 765	35 510	+ 1 150
7	19 "	28½	9 944	35 365	1 535	40 500	37 214	36 110	+ 745
8	19 "	28½	9 947	36 900		42 257	38 828	36 110	— 790
9	16 "	24	9 944	36 580	0	40 347	37 936	36 640	+ 60
10	16 "	24	9 944	36 580		40 347	37 936	36 640	+ 60
11	13 "	19½	9 952	36 857	343	39 361	37 758	37 090	+ 233
12	13 "	19½	9 947	37 200		39 728	38 110	37 090	— 110
13	10 "	15	9 955	36 480	83	37 946	37 008	37 460	+ 980
14	10 "	15	9 958	36 397		37 859	36 923	37 460	+ 1 063
15	7 "	10½	9 955	38 157	733	38 908	38 427	37 730	— 427
16	7 "	10½	9 942	38 890		39 657	39 166	37 730	— 1 160
17	4 "	6	9 947	49 500	1 740	49 818	49 614	37 910	— 11 590
18	4 "	6	9 949	51 240		51 570	51 359	37 910	— 13 330

Average of Column 8, for 1 to 16, inclusive = 38 000.


DIAMETERS IN LENGTH
BREAKING STRENGTH IN POUNDS PER SQ. IN.
WATERTOWN EXPERIMENTS
RANKINE'S FORMULA

shown by the tests made for the Cincinnati Southern Railway, but may be illustrated in a much more satisfactory manner from the experiments of Mr. Clarke and associates, which, being all made on columns built of the same iron, in the same shop, and tested in a machine of exceptional precision, may be considered as substantially free from the influences due to differences in the workmanship and manner of testing. The shape of the posts being also uniform, the principal factor of variation here is the length.

Column 7 of the accompanying table gives the values of f deduced from the results of experiments 1 to 18, inclusive, with Rankine's formula $f = \frac{P}{S} \left(1 + \frac{1}{36\,000} \frac{l^2}{r^2}\right).$

The gradual decrease of the values obtained, from 46 264 for post No. 1 to 38 908 for post No. 15, indicates that the value $\frac{1}{36\,000}$ assumed for a in the formula is too large. Column No. 8 gives the value of f corresponding to $a = \frac{1}{100\,000}$ $f = \frac{P}{S} \left(1 + \frac{1}{100\,000} \frac{l^2}{r^2}\right).$

These are remarkably uniform, the extreme variation from the average (38 000) being less than three per cent.

With this average of 38 000 assumed for f, and $a = \frac{1}{100\,000}$ in the formula, the calculated values of $\frac{P}{S}$ will agree very nearly with the actual results of the tests, as a comparison between Columns 5 and 9 of the table will show. It is remarkable indeed that the differences between the actual and calculated values of $\frac{P}{S}$ should be less than the differences between the resistances as given by two tests on similar posts. This is seen at a glance by a comparison of Columns 6 and 10 of the table.

The accompanying diagram illustrates further the close agreement between the formula and the experiments.

The sudden divergence noticeable for posts 17 and 18 is explained by the fact that they did not fail by bending, but by a buckling of the metal.

Discussion by Theodore Cooper, M. A. S. C. E.

The very careful tests made upon wrought iron columns of different forms under Mr. Bouscaren's direction, have been of great service to the profession. They drew attention to the defects existing in the best of our columns in use at that time (1875), and have produced a great improvement in the forms, proportions and workmanship of the columns now used by the more careful of our designers and builders.

It does not seem a matter of much doubt, that were similar tests made upon the columns now used, under as disinterested an examiner, we should have far better results upon all of our columns.

When it is appreciated that these were the first general tests made upon modern columns of large sizes, the great benefit to our practical knowledge becomes evident.

With the exception of a few isolated experiments, made by a couple of our bridge building firms for their own purposes, we had no tests later than those made by Hodgkinson for the Conway and Britannia tubular bridges.

The experiments upon Phœnix columns made at Watertown Arsenal, are also a valuable addition to our stock of knowledge upon well made columns of varying dimensions.

The well known Gordon's formula—

$$\frac{P}{S} = \frac{36\,000}{1 + \frac{l^2}{3\,000\,h^2}} = \text{crippling strain per square inch,}$$

which has been generally assumed as the standard of strength for all wrought iron columns with square ends, had its numerical constants derived from a series of experiments made by Mr. Hodgkinson upon small bars and plates, whose cross-section varied from one inch square to that of a rectangle $5\frac{3}{4}$ inches by one inch.

It was very reassuring to find from Mr. Bouscaren's experiments that a formula based upon such small specimens gave results as near as it did to the experimental ones.

Though the above formula was only intended to represent the strength of columns with a solid square or rectangular section, it has been generally applied to columns of all shapes of cross-sections.

The general form of the above formula was derived in the following manner :

The direct crushing strain upon any section of a column is represented by

$$\frac{P = \text{total load.}}{S = \text{Sectional area,}} \qquad (1.)$$

Any tendency to bend of a flexible column, free at the extremities, would increase the above crushing strain upon the concave side of the column, and relieve it upon the convex side. Analysis has shown that this increased strain due to the bending is theoretically equal to

$$\frac{P\,l^2\,\lambda}{\pi^2\,I} \text{ for the section at the centre of the column} = \frac{P\,l^2\,\lambda}{\pi^2\,S\,r^2} \qquad (2)$$

where P = total load on column, S = Sectional area,

l = length, I moment of inertia, and λ the compression per square inch for a units length. This is identical with the formula for limiting strength given by Euler for long flexible columns which fail solely by flexure $P = \pi^2 \dfrac{E\,I}{l^2}$, if we reduce by substituting for E, modulus of elasticity, its value $\dfrac{f'}{\lambda}$, f' being strain per square inch due to bending only.

Summing equations 1 and 2 for total strain f, and representing the numerical quantity $\dfrac{\lambda}{\pi^2}$ by the constant a' we get

$$\frac{P}{S} = \frac{f}{1 + a'\dfrac{l^2}{r^2}} \qquad (3) \qquad \text{Rankine's formula.}$$

Or for rectangular columns only

$$\frac{P}{S} = \frac{f}{1 + a'\dfrac{l^2}{h^2}} \qquad (4) \qquad \text{Gordon's formula.}$$

r being radius of gyration and h least side of rectangle.

From the experiments of Hodgkinson upon small bars and plates of wrought iron, Gordon determined the value of a to be $\frac{1}{3\,000}$ for square ended columns.

Rankine from the relation

$$\text{Mom. Inertia of a square} = \frac{S\,h^2}{12} = S\,r^2 \quad \text{or}$$

$$h^2 = 12\,r^2 \quad \text{or}$$

$$\frac{l^2}{3\,000\,h^2} = \frac{l^2}{36\,000\,r^2}$$ converted Gordon's factor into 36 000 for his own formula.

The numerical factors being obtained from the same experiments, both formulæ should be equally true for square or rectangular solid columns.

For any other form of column Gordon's formula can not be considered as applicable. Rankine's being of a more general form should be more nearly correct, and from the closer coincidence of the results obtained by Mr. Bouscaren, we are reassured of its greater correctness.

It, however, has not been generally used, no doubt from the apparent difficulty of obtaining the radius of gyration of various sections. This, however, can be readily overcome by substituting in the Rankine's form the average value for the radius of gyration of each particular form of cross-sections in terms of the least side or the diameter.

The numerical constants a and a' being derived from experiments on small bars, would very probably be different for columns of practical shapes and sizes.

Theoretically they represent a factor of the compression of the material, supposed fully elastic. Practically they represent a certain influence of the ultimate (not elastic) compression and extension, and also the influence of the fitting of the columns in the testing machine, including the squareness of the ends and comparative axial direction of the applied strains.

As the *ultimate* compression is not strictly a factor of the modulus of elasticity, but rather a factor dependent upon the quality of the iron as to ductility, we should expect the result pointed out by Mr. Bouscaren: that the *ultimate* resistance does not appear to be dependent upon the modulus of elasticity.

As it would appear to be relatively easier to bring the ends of large pieces to a square bearing, and the axis of strain more coincident with the axis of large columns than with the small bars experimented upon by Mr. Hodgkinson, we could reasonably expect in a new formula a smaller value for the constants a and a'. In examining and comparing experiments upon columns of different forms and makes, we must bear in mind the possible variations that will occur from practical considerations, some of which have been pointed out by Mr. Bouscaren.

1st. The proportion of parts, such as the relative thickness of the metal to the size of the cross-section ; (both Hodgkinson and Bouscaren's experiments show that a flat surface exceeding thirty times its thickness in breadth, will not be able to sustain its required compression without crippling)—the proper size and spacing of lattice bars, when used—the proportionate number of rivets at different parts of the column to resist the longitudinal shear due to the bending of the column or an unequal distribution of the pressure.

2d. The character of the iron as affects its resistance to crushing.

3d. The condition of the plates, channels or other forms as they come from the rolling mill ; and the necessary work to be put upon the pieces to take out the bends, warps and buckles that are almost always to be found in them. The fearful ordeal of the straightening sledge or drop leaves the pieces in a straighter condition, but at the expense of initial strains or a want of homogeneity in the resisting power of the metal.

4th. The character of the workmanship as shown in the riveting, the planing of the ends truly parallel and at right angles to the axis of the column, and the straightness of the column as a whole.

5th. The defects due to an aggregation of several pieces of possibly varying resistances to crushing and bending, thus producing a want of symmetry in the strength of the column. This being the probable explanation of the anomaly of columns like Nos. 13, 19, 21, 32 and 42 of Mr. Bouscaren's experiments, failing in their apparently strongest direction.

6th. The indefiniteness of the crippling point as determined by different observers, and the different manner of making the tests.

"The care taken to have the strains of compression pass exactly through the axis of the column" and "the ends brought to a good bearing," as noted by Mr. Howard, would have a very important influence upon the final result. The rapidity of making the test would also produce a variation in the recorded strength.

7th. The relative moduli of elasticity of the columns ; though the effect due to this factor would be more apparent in tests for limit of elasticity than for ultimate strength ; for as before remarked the elongations and compressions of the material at or near the crippling point would be more dependent upon the ductility than the modulus ; but within the elastic limit, the bending strain would undoubtedly be largely dependent upon the modulus of elasticity of the column.

The difficulty of recognizing the varying influences effecting the strength of columns of various forms and dimensions, compels us to the necessity of accepting *averages*, the reliability of which is largely dependent upon the number of our experiments.

Upon Diagram No. 1, Plate I, we have plotted with reference to the *ratio of lengths to diameters* all the most reliable experiments of Messrs. Hodgkinson, Bouscaren and Howard upon square-ended columns, with some few miscellaneous ones collected by the writer ; only omitting such ones as were plainly (from our present knowledge) defective in construction.

Upon Diagram No. 2, Plate II, we have plotted the same experiments with reference to the *ratio of the lengths to the least radius of gyration.* This being a much more reliable method of comparison of columns of various cross-sections ; for we not only eliminate the influence of the shape of the cross-section, but also have a definite dimension as a reference, which we have not in accepting the least side or diameter. For example, in a Phœnix column the nominal dimension used for a four segment column is the external diameter of the shaft, but for a greater number of segments the least dimension would include a portion of the flanges ; or if we still cling to the diameter of the shaft, why, in channel columns, should we not also accept the distance over the webs instead of over the flanges. However looked at, there is no positive comparison of different sections but by the radius of gyration, and we may consider that twice the radius of gyration is the " effective diameter " of any section.

The relation existing between the least dimension and the radius of gyration of the usual sections is given below :

TABLE No. 1.

Cross Section.	$r^2 : h^2$.	$r : h$.	Comparative values of a for use in Gordon's Formula.
Solid Rectangle......	$r^2 = \frac{h^2}{12}$	$r = \frac{h}{3.46}$	$\frac{1}{3000}$
Box Column.........	$r^2 = \frac{h^2}{6}$ approx.	" $= \frac{h}{2.45}$ approx.	$\frac{1}{6000}$
Open Col. of Channels.	$r^2 = \frac{h^2}{7.3}$ "	" $= \frac{h}{2.7}$ "	$\frac{1}{4930}$
Solid Circle..........	$r^2 = \frac{h^2}{16}$	" $= \frac{h}{4}$	$\frac{1}{2250}$
Hollow (thin) Circle...	" $= \frac{h^2}{8}$ approx.	" $= \frac{h}{2.82}$ approx.	$\frac{1}{4500}$
Phœnix Col..........	" $= \frac{h^2}{7.6}$ "	" $= \frac{h}{2.75}$ "	$\frac{1}{4750}$
Cross + ...	" $= \frac{h^2}{24}$ "	" $= \frac{h}{4.9}$ "	$\frac{1}{1500}$
Amer. Co.'s Col.......	" $= \frac{h^2}{11.6}$ "	" $= \frac{h}{3.4}$ "	$\frac{1}{3100}$

The fractions in the second member of the equations, under the third column, represent the comparative lengths of columns of different cross sections of equal strength. For example, the relative length of a Phœnix Column and an American Company's Column of the same strength would be as $\frac{1}{2.75} : \frac{1}{3.4}$ or as 34 diams. is to $27\frac{1}{2}$ diams.

This shows the importance of referring to the ratio of the radius of gyration instead of trying to compare by the ratio of diameters between columns of such unequal resisting capacity.

Returning, therefore, to diagram No. 2, as a fair comparative view of such tests as we possess, it will be seen that the diagram can be divided into three different zones marked *A*, *B* and *C*, which would appear to mark three different class of columns somewhat similar to that pointed out by Hodgkinson for cast iron columns.

In zone A, the resistance of columns from 1 to 50 radii of gyration seems to drop along an inclined strip bounded by lines running from

59 000—38 000
and 46 000—27 000

In zone B, from 50 to 120 radii of gyration, to be bounded by lines nearly horizontal, running from

37 000—35 150
and 26 000—25 000

In zone C, beyond 120 radii of gyration, the lines descend with a decreasing rate and converging as they approach the higher ratios.

This apparently anomalous result, of different laws of strength for columns of different lengths, noticed by Hodgkinson in cast iron columns, and partially visible in the Watertown experiments, is here clearly shown to be true of all the experiments upon square ended columns. If we could not determine the reason for this change of strength and correct our formulæ to provide for it, we would be justified in adopting "separate formulæ for long and for short columns," as is suggested by Messrs. Clarke, Reeves & Co., in their report of the Watertown experiments.

We will endeavor below to offer an explanation for the occurrence of the three zones of action; from which it will be seen that our previously accepted theoretical formula is defective in omitting a very important factor.

Zone A, comprised the limits wherein the columns have failed solely by crushing or failure by flexure cannot be expected. The writer believes the inclination of the crushing line to be simply due to the operator waiting for the same amount of visible crushing in a short as he would in a longer column.

Zone B, is at first inspection still more anomalous as it shows an almost equal strength for columns varying from about 20 diameters to 42 diameters in length, which would lead us to believe at the first view that this zone is also free from failure by flexure. Upon consideration it is *evidently due to the manner of testing the columns*, or in other words, our theoretical deductions are defective, *when applied to experiments made, as all of these were*, between two opposite and rigidly parallel faces of a testing machine.

Let Fig. 1 represent a square ended column being compressed between two rigidly parallel faces A and B of a testing machine. Upon any tendency of the columns to bend to the right as shown by the dotted lines, the centre of pressure is also transferred towards the right, instead of continuing to act through the axis; we thus get a movement resisting the tendency to bend, and which, within certain limits, will counteract the flexure that would otherwise cripple the column.

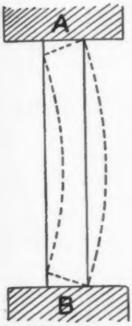


FIG. 1.

In very few of our practical applications of square ended columns could we expect this rigid maintenance of parallelism of the bearings, and therefore we would not be justified in assuming these tests as strictly applicable to practical cases.

The first part of test No. 3 of the Watertown experiments illustrates the principle above mentioned very clearly. This column was tested with hemispherical ends, one of which was found to be imperfect so that the bearing point was half an inch out of centre. After this column had been strained to about 20 000 lbs. per square inch, and had taken a permanent bend of +0.102 inches the eccentric head was reversed and the strain increased to 23 700 lbs.; this shifting of the centre of pressure removed the first permanent bend and gave it one of —.008 in the opposite direction. Thus clearly illustrating the effect of resisting or reversing the bending tendency, by any decentralizing action of the centre of pressure, when, as shown by Fig. 1, it is towards the same direction as the original flexure.

Zone C, comprises but a few experiments upon large columns, and hence cannot be considered so satisfactory as the results shown in A and B. We are, however, justified from what data we have, in believing that at some point at or beyond 120 radii of gyration the sections of the columns become so small in reference to the length that the resisting moment of the ends diminishes rapidly and reduces to nothing finally. Hence we should here expect, as shown by the experiments given in this zone, a descending curve represented by our theoretical formula

$$\frac{P}{S} = \frac{f}{1 + a\frac{l^2}{r^2}}$$

PIN-ENDED COLUMNS.

We have confined our remarks heretofore to square-ended columns exclusively, as the great mass of our experiments are upon the latter kind of columns.

It is to be regretted that we have not a more extended series of experiments upon pin-ended columns, for both the theoretical formulæ and the practical results of the tests we possess are unsatisfactory.

The formulæ for pin-ended columns

$$\frac{P}{S} = \frac{f}{1 + 4\,a\,\frac{l^2}{r^2}} \quad \text{.............. (Gordon's.)}$$

$$\frac{P}{S} = \frac{f}{1 + 2\,a\,\frac{l^2}{r^2}} \quad \text{..... ....(Bouscaren)}$$

would appear to be wrong as compared to the usual one for square-ended columns ; for at very high ratios the effect of the ends should be identical, as the square ends would be practically reduced to round ends. The formulæ, however, give results for the high ratios, 2 to 4 times as great for square-ended columns as for the pin-ended ones ; instead of the lines of strength converging towards the higher diameters, they are constantly diverging, which does not appear reasonable.

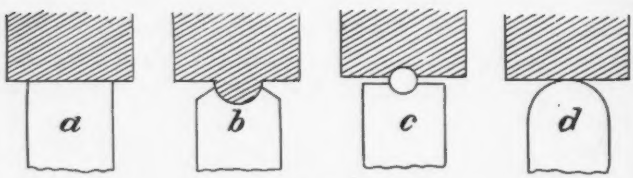


FIG. 2.

Practically, the writer believes, the strength of pin-ended columns would be found to be somewhat governed by the diameters of the pins (aside from the crushing effect upon the bearings) and the closeness of their fit to the pin hole. For the same resisting action against bending, as has been mentioned, under square-ended columns would, no doubt, here occur to the extent of the frictional moment about the pin. The round-ended column *d*, is the only one of those sketched in Fig. 2 which would be free from this resistance to flexure.

If we possessed a sufficiently extended series of experiments upon round, pin and square-ended columns, we would probably find that those on round ends gave the closest correspondence to our theoretical formula, and the others could be derived from it by adding a factor representing the resisting influence of the ends. Such a formula would not only be rational, but would point out more definitely the effects of the ends than those we now have in use; where as much weight is given to the squareness of the ends upon a column 100 diameters long as to a column 10 diameters.

As it may be far in the future that we can possess such a complete set of experiments, we are justified in attempting temporary formulæ to represent the present state of our knowledge.

Temporary Formulæ for Wrought Iron Columns.—It will appear plain to those who examine our diagrams, that when we consider only the range of the zones A and B, the result of the experiments can be represented by numerous formulæ as long as we introduce proper constants. But in order for our formulæ to approximate to the truth, we must extend their range over the whole field of our experiments.

Square-Ended Columns.—From our remarks upon the effect of the squareness of the ends of the columns to prevent the bending tendency, we may consider the effect to be equivalent to a certain reduction of the effective length of the column or of the ratio $\frac{l}{r}$

If we therefore represent the ratio $\frac{l}{r}$ by R in our formula and call $m\frac{l}{r} = n$ the proportionate reduction of the total ratio by the effect of the squareness of the ends, our theoretical formula would now be

$$\frac{P}{S} = \frac{f}{1+a\ (R-n)^2} \qquad (5)$$

Applying this formula to the experiments plotted on Diagram No. 2, we get the following numerical values: $f = 36\,000$, $n = 80$, $a = \frac{1}{18\,000}$ or

the following formula : $\frac{P}{S} = \frac{36\,000}{1 + \frac{(R-80)^2}{18\,000}}$ .... (a)

as representing very closely the experiments on Phœnix columns.

The minimum curve of the experiments upon large sized columns would appear to be represented very closely by the formula

$$\frac{P}{S} = \frac{30\,000}{1 + \frac{(R-80)^2}{18\,000}} \quad \ldots\ldots\ldots\ldots \quad (b.)$$

These two formulas are plotted upon this diagram, and their coincidence with the experiments would seem to justify us in accepting them for all columns where the ends *are held rigidly square,* as were those in the testing.

For Pin Ended Columns we should have, if our claim previously made in regard to the resisting moment of the friction is correct, a formula like equation (5), but with a lower value of n.

For Round Ended Columns, where there is no resisting moment, the same equation (5) should apply, by making $n = o$.

In the following table we have determined the value of f for the different classes of columns from the following formulae:

Square Ended Columns $\frac{P}{S} = \frac{f}{1 + \frac{(R-80)^2}{18\,000}}$ .... (c.)

Pin Ended Columns $\frac{P}{S} = \frac{f}{1 + \frac{(R-33)^2}{18\,000}}$ .... (d.)

Round Ended Columns $\frac{P}{S} = \frac{f}{1 + \frac{R^2}{18\,000}}$ .... (e.)

TABLE No. 2.

VALUES OF CONSTANT f DERIVED FROM EXPERIMENTS BY USE OF FORMULÆ c, d AND e.

EXPERIMENTS MADE BY.	Kind of Column.	Form of Bearings.	Ratios. $\frac{l}{d}$	Ratios. $\frac{l}{r}=R$	Formulæ used for f.	Strain per Square Inch by Experiment.	Value of f as derived from Experiment.	
Mr. Bouscaren........................	Phœnix.	Square.	22.4	61.6	(c.)	37.500	36.800	
"	"	"	39.9	110	"	31.000	32.550	
"	"	"	40.7	112	"	34.800	36.780	
"	"	"	40.7	112	"	36.600	38.680	
Mr. Howard........................	"	"	42*	115½	"	34.650	37.000	*Average of two experiments.
"	"	"	37½*	103	"	35.155	36.180	
"	"	"	33*	91	"	34.965	35.200	
"	"	"	27½*	76	"	36.130	36.120	
"	"	"	24*	66	"	36.580	36.180	
"	"	"	19½*	54	"	37.030	35.680	
"	"	"	15*	41	"	36.440	33.750	
"	"	"	25	69	"	36.010	35.750	
Phœnix Iron Company..................	"	"	53.5	147	"	30.274	37.820	
" "	"	"	35.9*	98.7	"	37.900	38.630	
Average value of f for square ended	columns.	....	....	....	....	....	36.220	From 22 columns.
Mr. Bouscaren........................	Phœnix.	Round.	39.9	110	(e.)	21.700	36.280	
Phœnix Iron Company..................	"	"	53.6	147	"	16.387	36.120	
Average value of f for round ended	columns.	....	....	....	....	....	36.200	From 2 columns.

These averages from 24 Phœnix columns, varying from 15 to 53.5 diameters in length (41 to 147 radii of gyration), cover values of f extending from 32.550 to 38.680, or a variation from the average of only 10 per cent.

TABLE No. 3.

EXPERIMENTS MADE BY.	Kind of Columns.	Form of Bearings.	RATIOS.		Formulæ used for f.	Strain per Square Inch by Experiment.	Value of f as derived from Experiment.	
			$\frac{l}{d}$	$\frac{l}{r}=R$				
Mr. Bouscaren........................	Amer. Co.'s.	Square.	25.3	81.6	(c)	31.500	31.500	
"	"	"	32.4	110	"	27.800	29.190	
"	"	"	45	155	"	23.700	31.106	
"	"	Pin.	24	81.3	(d.)	26.500	29.935	
"	"	"	29	101	"	24.000	30.165	
"	"	"	30	102.5	"	26.700	33.860	
"	"	"	31.2	106	"	22.000	28.510	
General average for both square and pin ended columns.....			....	....	....	....	30.609	From 7 experiments.

Greatest variation being from 28.510 to 33.860, or about 11 per cent. from the average.

TABLE No. 4.

EXPERIMENTS MADE BY.	Kind of Column.	Form of Bearings.	RATIOS. $\frac{l}{d}$	$\frac{l}{r}=R$	Formulæ used for f.	Strain per Square Inch by Experiment,	Value of f as derived from Experiments
Mr. Bouscaren.	Box Col's.	Square.	35	98	(c.)	30.200	30.740
"	"	"	34.1	84	"	33.200	33.246
"	"	"	41.6	102	"	30.000	30.810
"	"	Pin.	30.9	93	(d.)	25.500	30.600
General average..	....	....	....	....	... .	....	31.600

Greatest variation 30.600 to 33.246, or 5 per cent. from average.

TABLE No. 5.

EXPERIMENTS MADE BY.	Kind of Column.	Form of Bearings.	RATIOS. $\frac{l}{d}$	$\frac{l}{r}=R$	Formulæ used for f.	Strain per Square Inch by Experiment.	Value of f as derived from Experiments
Mr. Bouscaren.......	Op'n Col's	Square.	23	60	(c.)	32.300	31.580
"	"	"	27.5	73	"	29.600	29.520
"	"	"	27.5	72	"	32.400	32.300
"	"	"	33	90	"	32.300	32.480
"	"	Pin.	45.5	130	(d.)	18.000	27.410
"	"	"	51	138	"	23.128	37.294
"	"	"	34¼	76	"	31.700	34.940

Average for square ended columns = 31.470. Variation from average is only 6 per cent. For the pin ended columns the variation is much greater, and especially between the two columns of nearly same number of radii in length, which would relieve the formula of the blame. As these three columns were of different forms and material, made at different shops, and tested at different places, we can consider that the want of correspondence to our formulæ, which have been so close for so many other columns, is rather due to other circumstances, perhaps largely to the relative closeness of the fit of the pins.

TABLE No. 6.

Experiments Made by	Kind of Column.	Form of Bearings.	Ratios.		Formulæ Used c or f.	Strain per Square Inch by Experiment.	Value of f as Derived from Experiment.
			$\frac{l}{d}$	$\frac{l}{r} = R$			
Mr. Bouscaren......	Keystone.	Square..	21.7	60	(c)	30.000	29.330
	"	"	20.3	64	"	32.000	31.540
	"	"	37.6	135	"	27.800	32.470
	"	"	34.1	96	"	30.000	30.426
	"	"	20	56	"	36.900	35.720
	"	Round...	35.1	98	e	22.000	33.738

General average equals, 32.200, with a maximum variation from the average of 11 per cent.

No doubt, a more extended series of experiments upon the different classes of columns, would enable us to adopt constants still more exactly representing the strength of these columns. And, as before remarked, columns, as now made, with the light thrown upon their weak points by Mr. Bouscaren's experiments, would undoubtedly give better results than those recorded above. It would also be reasonable to expect that the value of a would vary with different classes of irons as well as that of the constant f.

Fron the result of our comparison, it seems proper to assign to f in formulæ (c) (d) and (e) the following values :

For Phœnix columns, $f = 36.000$
" Amer. Co.'s " $f = 30.000$
" Box and open " $f = 31.000$

Formulæ c, d, and e, as before explained, can be reconverted into formulæ containing the ratio $\frac{l}{d}$ instead of $\frac{l}{r}$ by the use of our previous table No. 1, but the formulæ then must be applied to a special form of column only, instead of being general as are c, d, and e.

Calling $\frac{l}{d} = H$, and using the values from Table 1, we get the following special formulæ:

For *Phœnix Columns* :

$$\left.\begin{aligned}
&\text{Square Ends,}\quad \frac{P}{S} = \frac{36\,000}{1+\frac{(H-30)^2}{2\,400}}\\
&\text{Pin Ends,}\quad \text{“} = \frac{36\,000}{1+\frac{(H-12)^2}{2\,400}}\\
&\text{Round Ends,}\quad \text{“} = \frac{26\,000}{1+\frac{H^2}{2\,400}}
\end{aligned}\right\}(6)$$

For Amer. Co.'s Columns :

$$\left.\begin{aligned}
&\text{Square Ends,}\quad \frac{P}{S} = \frac{30\,000}{1+\frac{(H-24)^2}{1\,600}}\\
&\text{Pin Ends,}\quad \text{“} = \frac{30\,000}{1+\frac{(H-10)^2}{1\,600}}\\
&\text{Round Ends,}\quad \text{“} = \frac{30\,900}{1+\frac{H^2}{1600}}
\end{aligned}\right\}(7)$$

For Box or Square Columns :

$$\left.\begin{aligned}
&\text{Square Ends,}\quad \frac{P}{S.} = \frac{31\,000}{1+\frac{(H-32)^2}{3\,000}}\\
&\text{Pin Ends,}\quad \frac{P}{S} = \frac{31\,000}{1+\frac{(H-13)^2}{3\,000}}
\end{aligned}\right\}(8)$$

For Open Columns of Channels :

$$\left.\begin{aligned}
&\text{Square Ends,}\quad \frac{P}{S} = \frac{31\,000}{1+\frac{(H-30)^2}{2\,475}}\\
&\text{Pin Ends,}\quad \frac{P}{S} = \frac{3\,1000}{1+\frac{(H-12)^2}{2\,475}}
\end{aligned}\right\}(9)$$

NOTE: The value of f has no relation to the crushing strength of a short column, as is often claimed for this factor in Gordon's formula. It represents here what it did in Gordon's formula, a *numerical constant*, to take the place of the factor f in our theoretical formula—f being theoretically the greatest strain upon any unit of metal from the applied load.

LIMIT OF ELASTICITY.

In examining into the results of the experiments with reference to the elastic limits, we find a much greater variation than is shown by the crippling strain. This we should expect in columns containing any of the before mentioned defects of material or workmanship. The full influence of these defects would be visible in tests for the limit of elasticity, but only partially in those for crippling stress as the flow of the metal would accommodate itself somewhat to any irregularities after the elastic limit was passed.

The results given for the elastic limits in the tests made at Watertown, do not appear to the writer to be justified from the records as given. It is not clear to him what rule was adopted as the guide to locate this point. By an analysis of the data in those experiments, where the detailed compression is fully recorded, the elastic limit would appear to be much lower than that given in the tables. It is a difficult matter to determine the elastic limit in compression experiments, but it is better to err by placing it too low than too high.

Taking Mr. Bouscaren's experiments as more general, containing a greater variety of columns, and more nearly representing the cruder tests which columns would receive in actual structures, we have determined the values of f_1, corresponding to the limits of elasticity by the formulæ *c*, *d*, and *e*.

TABLE No. 7.

VALUES OF f_1, IN FORMULÆ c, d, AND e, FOR THE LIMIT OF ELASTICITY.

KIND OF COLUMN.	$\frac{l}{r} = R$	f_1.	
Keystone.	56	14 500	Average value of f_1, for limit of elasticity = 17 740 lbs.
	60	17 100	Minimum value, = 12 170 lbs.
	64	18 740	
	96	12 170	
	98	22 940	
	135	21 000	
American Co.'s	81.3	13 470	Average value of f_1, = 18 686 lbs.
	81.6	23 000	Minimum " " " = 13 470 " =
	101.	15 080	45 % of f the coefficient of crippling.
	102.5	18 960	Variation of minimum from average
	106	18 250	= 28 %
	110	24 940	
	155	17 100	
Square or Box.	84	14 960	Average value of f_1, = 16 330 lbs.
	93	18 670	Minimum " " = 14 960 = 48 % of f
	98	15 370	the coef. of crippling.
	102	16 320	Variation of minimum from average = 9 %
Open.	60	22 420	Average value of f_1, = 21 588
	72	23 000	Minimum " " = 18 270 =
	76	23 100	59 % of coef. of crippling.
	90	21 150	Variation of min. from average = 18 %
	130	18 270	
Phœnix.	110	18 880	Average value of f_1, = 22 355 lbs.
	110	23 170	Minimum " " = 18 880 " =
	112	18 950	52 % of f the coef. of crippling.
	112	28 420	Variation of min. from average = 18 %

Allowed Working Strain.

The determination of the allowed working strain per square inch of cross section is of vastly more interest for constructive purposes than the breaking or crippling strength.

The capacity of any material to resist repeated and continued loadings, is dependent upon restricting the maximum strains within the elastic limits; especially where the member forms part of a skeleton structure, in which the relative strains are entirely controlled by relative elongations and extensions of the several pieces.

We are not justified, however, in loading our columns up, or near to the elastic limit, for the following reasons:

1st. Our tests are merely *statical* tests with a limited number of loadings; also, our calculations of strains are all *statical;* while the actual loads applied in practice and the strains induced from them are *dynamic* in action and repeated in application.

2d. Columns beside being strained by direct loadings, are seldom exempt from the liability of being struck *transversely* by passing bodies.

3d. There should be some allowance for possible defects in workmanship and material.

It would therefore appear proper to limit the *maximum* allowed strain per square inch to within one-half of the least elastic limit of the columns.

Believing that the reference of the so-called "Factor of Safety" to the rupturing or crippling point is false in doctrine, and delusive in its effect, not only upon the general public, but, unfortunately, upon some of our own profession, the writer would suggest the greater correctness of considering the limit of elasticity as the proper point of reference. His own preference, however, is to determine as near as may be, the maximum strain to be allowed in each case for the required material, and then to specify the *maximum allowed strain*, instead of depending upon a fictitious factor of safety. Instead, therefore, of computing the crippling strength of a column by the preceding formulæ, and taking $\frac{1}{4}$, $\frac{1}{5}$ or $\frac{1}{6}$ of this as the allowed strain, he prefers to change the formulæ so they will give directly the allowed strain.

Taking 44 % (about the proportion used in tension members), of the

minimum value of f_1, from Table 7, we get the following values for f_2, the coefficient of allowed working strain :

American Co.'s columns,	f_2	=	5 930
Box	"	"	6 580
Open	"	"	8 040
Phœnix	"	"	8 310

That for American Co's columns is too low comparatively, for this minimum is from a pin ended column which gave a low value for rupture also, and the fault may have been due to a bad fitting of the pin. And as the general average for this class of columns is even higher than the average for the box columns, it would be proper to give the American Co.'s columns a higher figure than that derived from this exceptionally low minimum. Taking, therefore, the next higher value of f, we get for American Co's columns, $f_2 = 6\ 635$.

By introducing these values of f_2 in the general equations c, d and e, or in (6), (7), (8) and (9), we obtain equations giving the safe strain upon the several kinds of columns under loads acting upon the extremities of the columns similarly to the manner of testing the columns in the testing machines. While columns proportioned by such formulæ would be of equal strength comparatively for this kind of a loading ; they would not have equal resistance to forces acting transversely to the axes of the columns, and seldom would we be justified in neglecting a consideration of such transverse forces.

The columns with the higher coefficients would, for same loading equal lengths be of a smaller diameter than those of the lower coefficients, and hence more liable to deflection by side blows.

The longer diameters of each class of columns would also be of a decreasing strength against such an application.

Mr. C. Shaler Smith has partially provided for this difficulty by adopting an increasing factor of safety in the following manner :

$$\text{Factor of safety} = 4 + \frac{5}{100} H$$

which gives the following values for this factor :

$H = 10$	Factor of safety	$= 4.5$
20	"	$= 5.$
30	"	$= 5.5$
40	"	$= 6.$
50	"	$= 6.5$
60	"	$= 7.$

Believing this reduction to be very desirable, we will adopt the principle and reduce our values of f_2 in the same proportion, or as :

$$1 : 1 + 0.0125\ H,$$

and nearly as :

$$1 : 1 + 0.033\ R.$$

This reduces our general formulæ *c*, *d*, and *e* to the following form for *allowed working strain :*

$$\left.\begin{array}{lll}
\text{Square ends,} & \dfrac{P}{S} = \dfrac{f_2}{1 + \dfrac{(R-80)^2}{18\,000}} \div (1 + 0.033\ R) \\
\text{Pin ends,} & \text{"} = \dfrac{f_2}{1 + \dfrac{(R-33)^2}{18\,000}} \div \quad \text{"} \\
\text{Round ends,} & \text{"} = \dfrac{f_2}{1 + \dfrac{R^2}{18\,000}} \div \quad \text{"}
\end{array}\right\} \quad (10)$$

Where $\frac{P}{S}$ =allowed working strain per square inch, $R = \frac{l}{r}$ = ratio of length to least radius of gyration, f_2 = coef. of working strain.

For American Co.'s columns,	$f_2 = 6\,600$		
Box	"	"	$= 6\,500$
Open	"	"	$= 8\,000$
Phœnix	"	"	$= 8\,300$

To obtain similar formulæ in terms of the ratio of length to diameter, or least dimensions, we make the same substitutions in Formulæ (6), (7), (8) and (9).

For Phœnix columns :

$$\left.\begin{array}{lll}
\text{Square ends,} & \dfrac{P}{S} = \dfrac{8\,300}{1 + \dfrac{(H-30)^2}{2\,400}} \div (1 + 0.0125\ H) \\
\text{Pin} \quad \text{"} & \text{"} = \dfrac{8\,300}{1 + \dfrac{(H-12)^2}{2\,400}} \div \quad \text{"} \\
\text{Round} \quad \text{"} & \text{"} = \dfrac{8\,300}{1 + \dfrac{H^2}{2\,400}} \div \quad \text{"}
\end{array}\right\} \quad (11)$$

For American Co.'s columns:

$$\left.\begin{array}{ll}\text{Square ends, } \dfrac{P}{S} = \dfrac{6\,600}{1+\dfrac{(H-24)^2}{1\,600}} \div (1+0.0125\ H) & \\ \text{Pin \quad `` \quad ``} = \dfrac{6\,600}{1+\dfrac{(H-10)^2}{1\,600}} \div \quad \text{``} & \end{array}\right\} \quad (12)$$

For box or square columns:

$$\left.\begin{array}{ll}\text{Square ends, } \dfrac{P}{S} = \dfrac{6\,500}{1+\dfrac{(H-32)^2}{3\,000}} \div (1+0.0125\ H) & \\ \text{Pin \quad `` \quad ``} = \dfrac{6\,500}{1+\dfrac{(H-13)^2}{3\,000}} \div \quad \text{``} & \end{array}\right\} \quad (13)$$

For open columns:

$$\left.\begin{array}{ll}\text{Square ends, } \dfrac{P}{S} = \dfrac{8\,000}{1+\dfrac{(H-30)^2}{2\,475}} \div (1+0.0125\ H) & \\ \text{Pin \quad `` \quad ``} = \dfrac{8\,000}{1+\dfrac{(H-12)^2}{2\,475}} \div \quad \text{``} & \end{array}\right\} \quad (14)$$

where $H = \dfrac{l}{h}$ = ratio of length to least dimension.

In Table 8 we give the values of the preceding formulæ with and without the reducing factor $(1+0.0125\ H)$ for the several columns. and also of Gordon's formula with a factor of $\frac{1}{5}$.

TABLE No. 8.

Allowed Working Strain for Wrought Iron Columns.

SQUARE ENDED.

Ratio of length to Diameter.	Gordon's Formula, *f* lbs. per square inch.	Phœnix Columns.		American Co.'s Columns.		Box Columns.		Open Columns.		Remarks.
		a	b	a	b	a	b	a	b	
10	6 968	9 960	8 853	7 521	6 686	7 790	6 924	9 542	8 482	Column a gives the values obtained by omitting the factor $(1 + 0.0125\,H)$. Column b gives values as obtained by the full formulae 11–14.
20	6 353	8 661	6 928	6 666	5 333	6 828	5 462	8 337	6 670	
30	5 540	8 300	6 037	6 455	4 700	6 510	4 735	8 000	5 818	
40	4 696	7 968	5 312	5 700	3 800	6 364	4 243	7 690	5 126	
50	3 928	7 115	4 380	4 640	2 855	5 866	3 610	6 540	4 024	
60	3 272	6 037	3 450	3 646	2 084	5 153	2 945	5 867	3 352	
70	2 734	4 980	2 656	2 842	1 516	4 388	2 340	4 859	2 591	
80	2 300	4 065	2 032	2 230	1 115	3 676	1 838	4 000	2 000	
90	1 946	3 340	1 572	1 773	834	3 064	1 442	3 260	1 534	
100	1 660	2 730	1 213	1 432	635	2 560	1 140	2 685	1 190	

TABLE No 8 (*Continued*).

Allowed Working Strain for Wrought Iron Columns.

PIN ENDED.

Ratio of length to Diameters.	Gordon's Formula, ¼ lbs. per square inch.	Phœnix Columns.		American Co.'s Columns.		Box Columns.		Open Columns.		"Bouscaren's," Gordon's Formula, ¼ lbs. per square inch.	Remarks.
		a	b	a	b	a	b	a	b		
10	6 353	8 314	7 390	6 600	5 867	6 520	5 797	8 012	7 122	6 750	
20	4 695	8 250	6 600	6 212	4 970	6 395	5 116	7 798	6 238	5 684	
30	3 273	7 313	5 318	5 280	3 840	5 929	4 312	7 074	5 145	4 500	
40	2 300	6 256	4 170	4 224	2 816	5 229	3 486	6 075	4 050	3 484	
50	1 662	5 182	3 189	3 300	2 031	4 463	2 746	5 052	3 109	2 700	
60	1 241	4 234	2 419	2 575	1 471	3 743	2 139	4 143	2 367	2 117	
70	956	3 456	1 843	2 030	1 082	3 120	1 664	3 391	1 808	1 688	
80	755	2 836	1 418	1 615	808	2 604	1 302	2 789	1 399	1 367	
90	610	2 348	1 105	1 320	621	2 134	1 028	2 313	1 088	1 125	
100	500	1 963	872	1 088	484	1 845	820	1 937	861	940	

The great difference in the bending effect shown by the factors 1 600 to 3 000 as derived from the moments of inertia of the columns ; and as is shown in the tabulated figures ; illustrates the economy of massing the metal far from the axis of the columns, as is done in the Phœnix, Box and open columns.

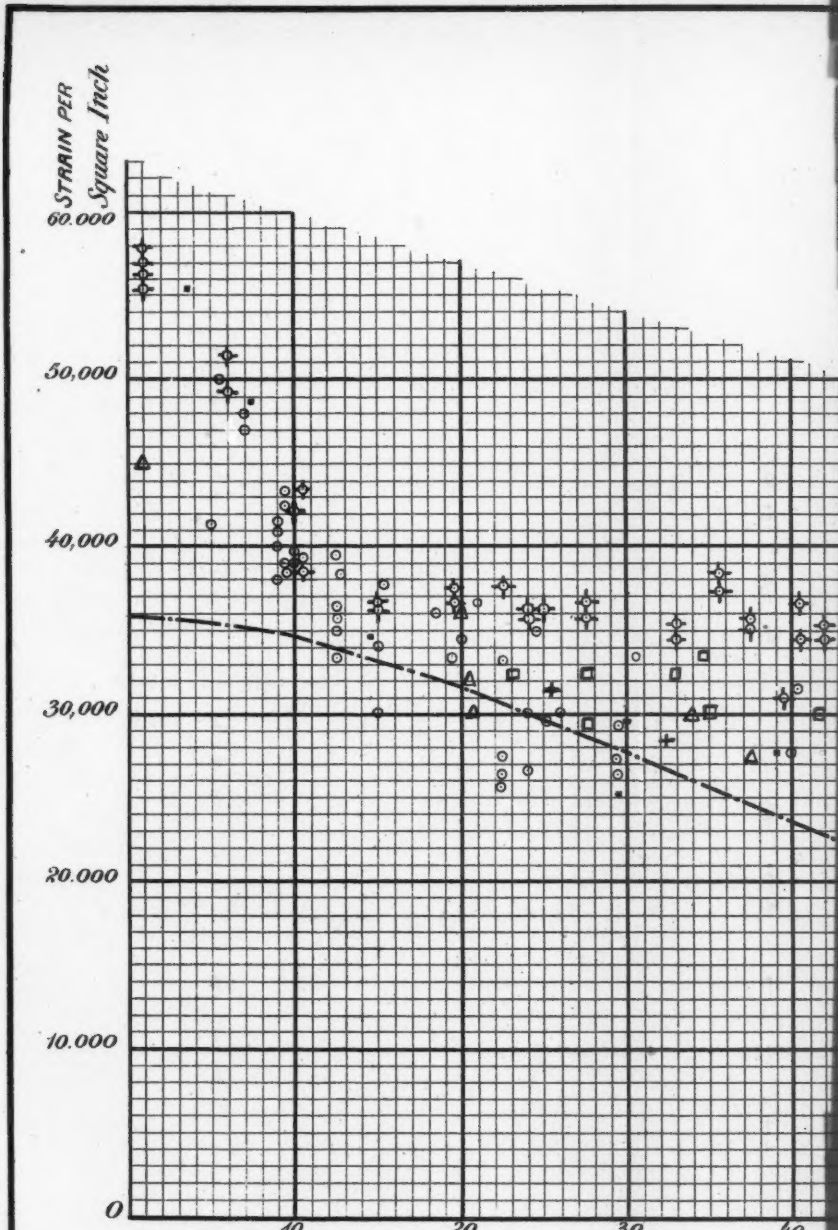
These figures, however, do not show the relative economy of the columns, for in the box and open columns there is a greater freedom in selecting the diameters, without increasing the areas proportionately, than in the Phœnix or Amer. Co.'s style of columns. Other factors, as the price of the material and facility of forming connections, also enter into the problem. We regret that Mr. Howard should have gone outside of the record, in his report upon Phœnix columns, to make the following remark: * * * "Therein they differ materially from lattice columns. The latter form (lattice columns), after deflecting slightly, suddenly give way by tearing out the riveting of the lattice bars, after which but little strength, as a column, remains." We fail to find any such action recorded in Mr. Bouscaren's experiments, and are not aware of any experiments upon well made and well proportioned lattice columns that would justify such a remark. Mr. Howard's position should make him cautious, to prevent his interest in a series of experiments from drawing him into any appearance of bias or partisanship.

Phœnix columns can well stand upon their excellent record and recognized merits.

Conclusion.—After a careful consideration of the experiments we possess, it becomes very clear that we cannot allow Gordon's formula to be our standard of strength for all forms and kinds of columns. Neither does Rankine's formula represent correctly the results of our tests.

Those given by the writer are submitted for the consideration of the profession.

But, whatever formula we may adopt, it has been made clear by the experiments of both Hodgkinson and Bouscaren, that the strength of wrought-iron columns are not only dependent upon the ratios of length to diameters, and the shape of the cross section ; but also to a very great extent upon the proportions of parts, details of design and workmanship, and the material from which the columns are made. So that it becomes as necessary in our specifications to detail carefully the kind of material, proportions of parts, and character of workmanship, as to specify the formula to be used in estimating the sectional areas.



STRAIN PER
Square Inch
60.000
50,000
40,000
30,000
20.000
10.000
0
10
20
30
40
Ratio of Length to diameters

EXPE

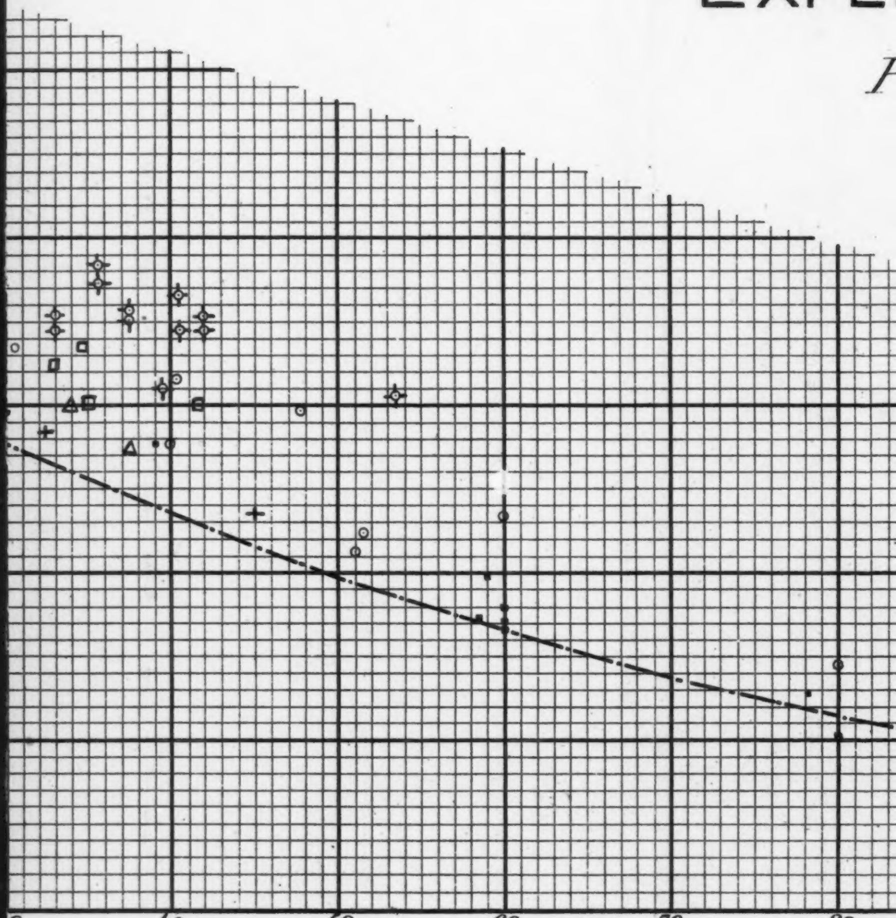


DIAGRAM No. 1.

PERIMENTS ON SQUARE-ENDE

Plotted by ratios of lengths to least dimensions

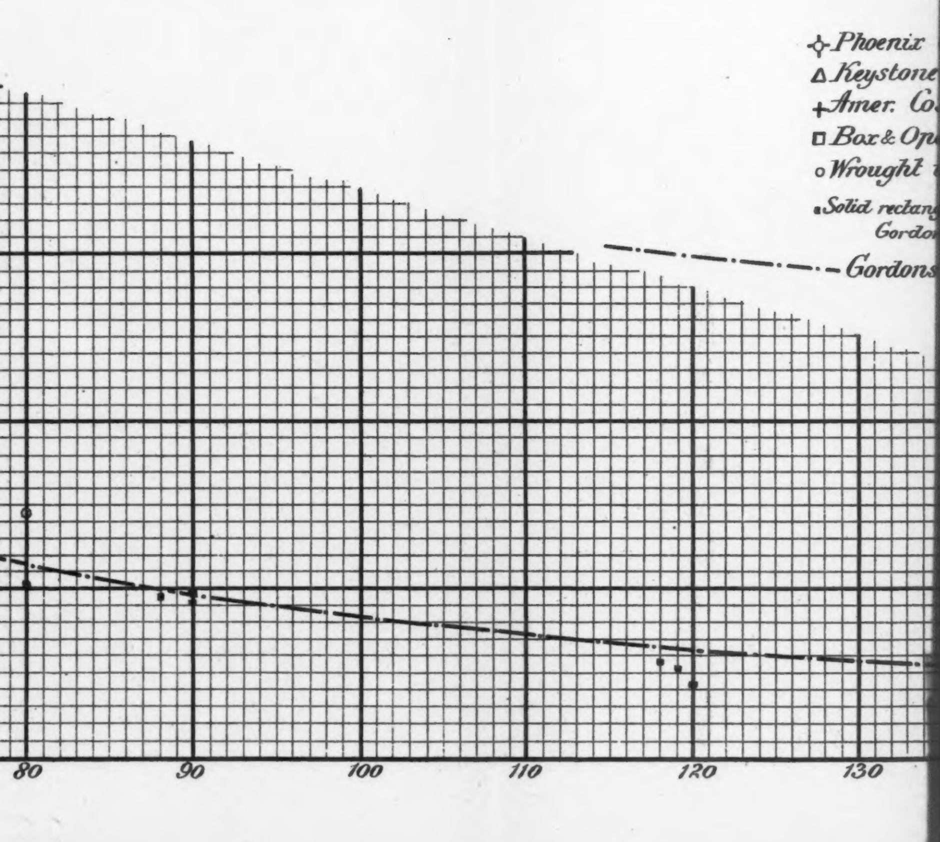


DIAGRAM No. 1.

EXPERIMENTS ON SQUARE-END

Plotted by ratios of lengths to least dimensi

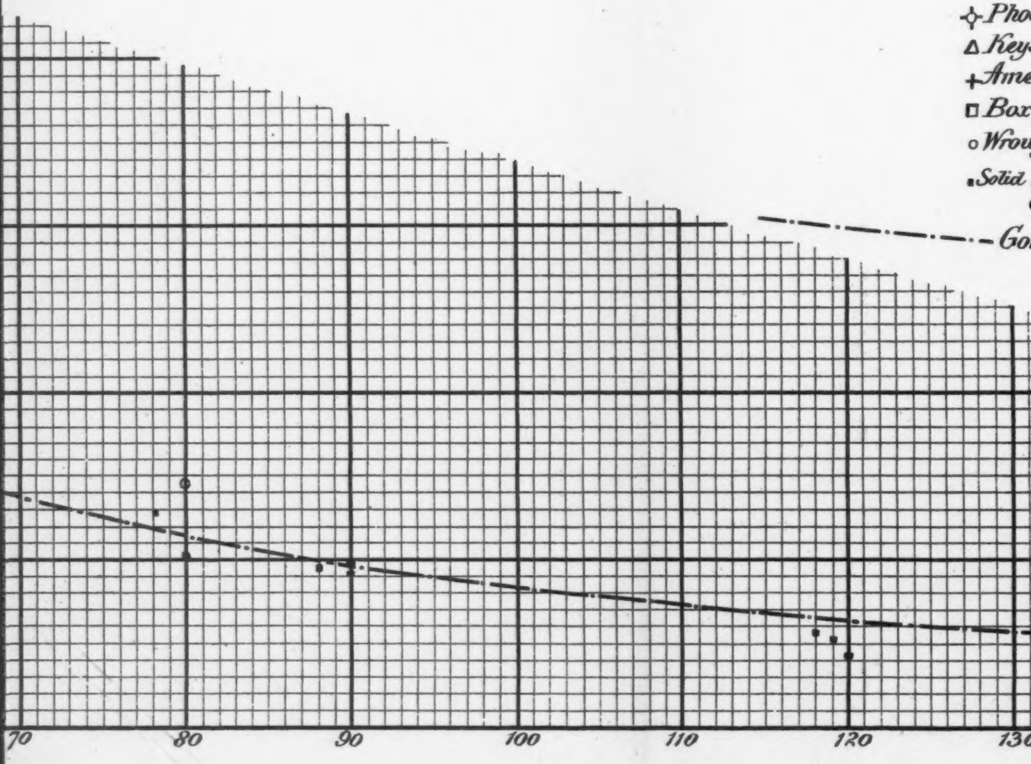
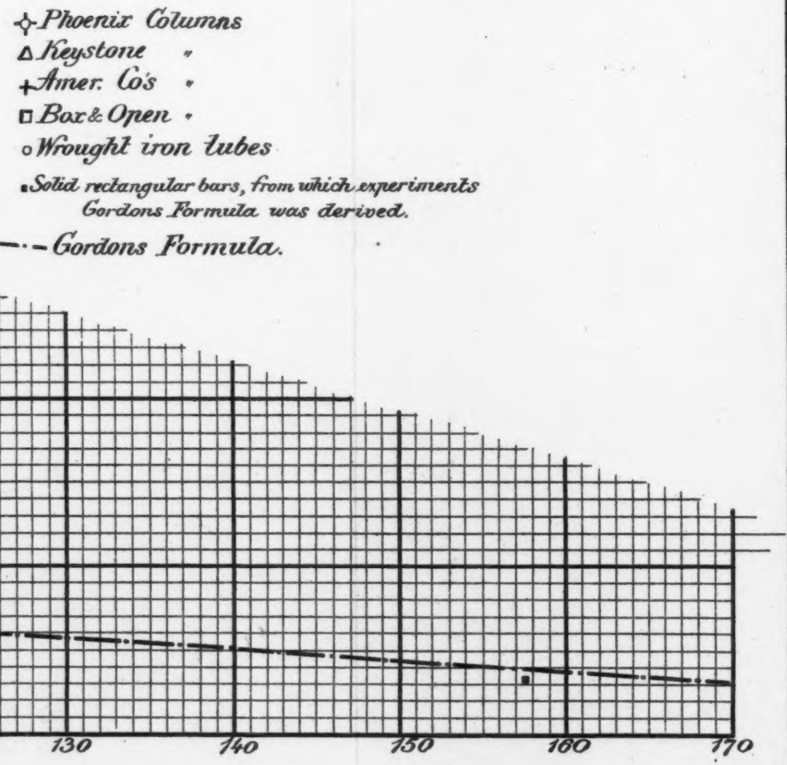
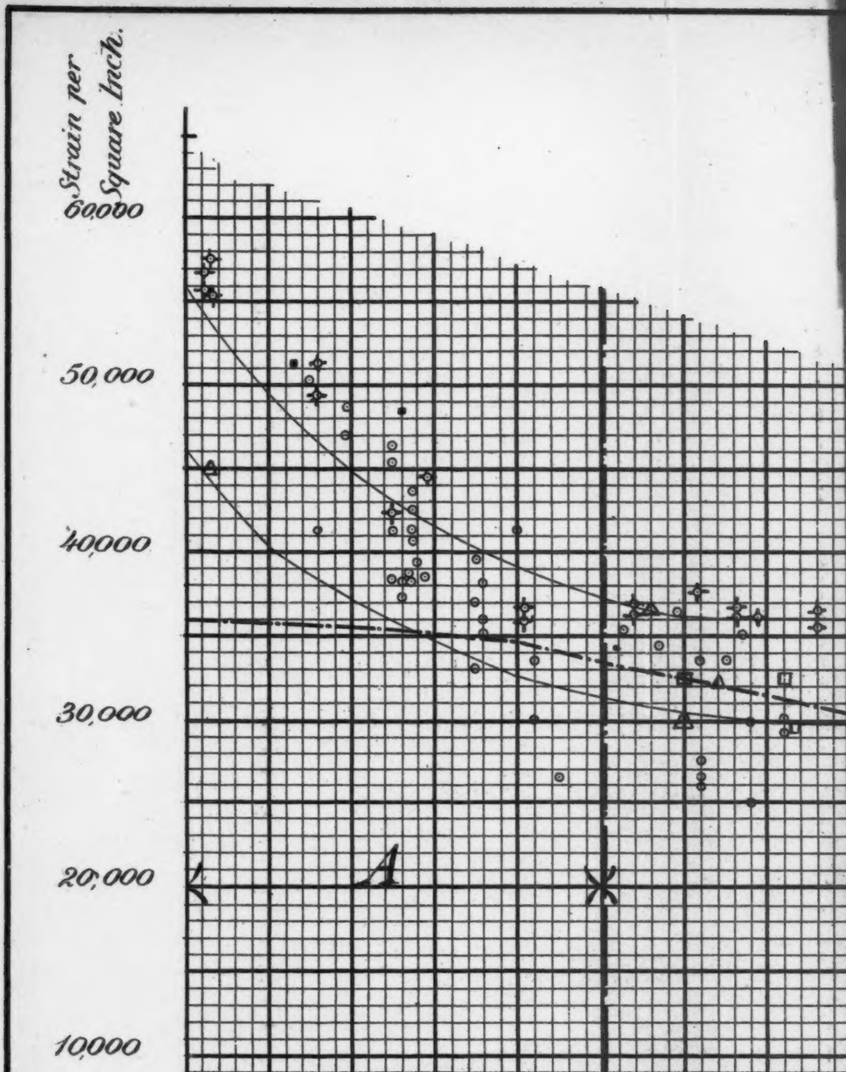


PLATE 1.
TRANS. AM. SOC. CIV. ENG'R'S
VOL. XI. Nº CCXXXIII.
COOPER
ON WROUGHT IRON COLUMNS.

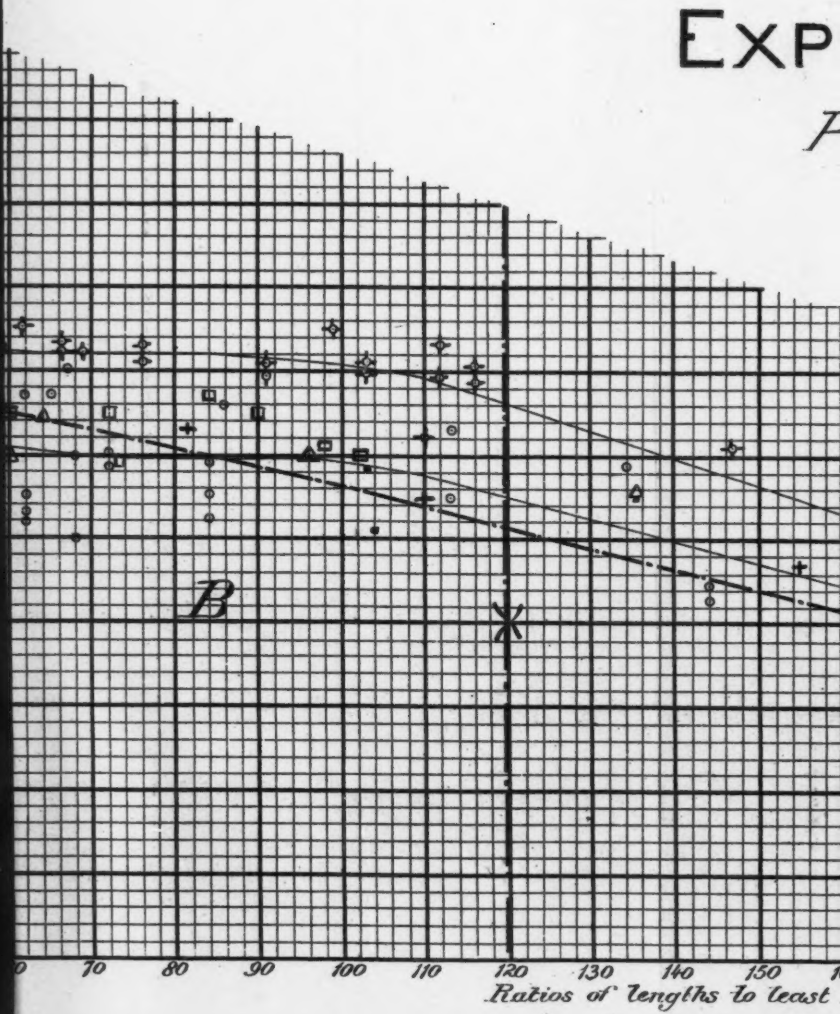
NDED COLUMNS

nensions $\frac{l}{h}=H$.





Strain per
Square Inch.
60,000
50,000
40,000
30,000
20,000
10,000
0.
A
0
10
20
30
40
50
60
70



EXP
B
70
80
90
100
110
120
130
140
150
Ratios of lengths to least

DIAGRAM No. 2

…XPERIMENTS ON SQUARE-…

Plotted by ratios of length to least rad…

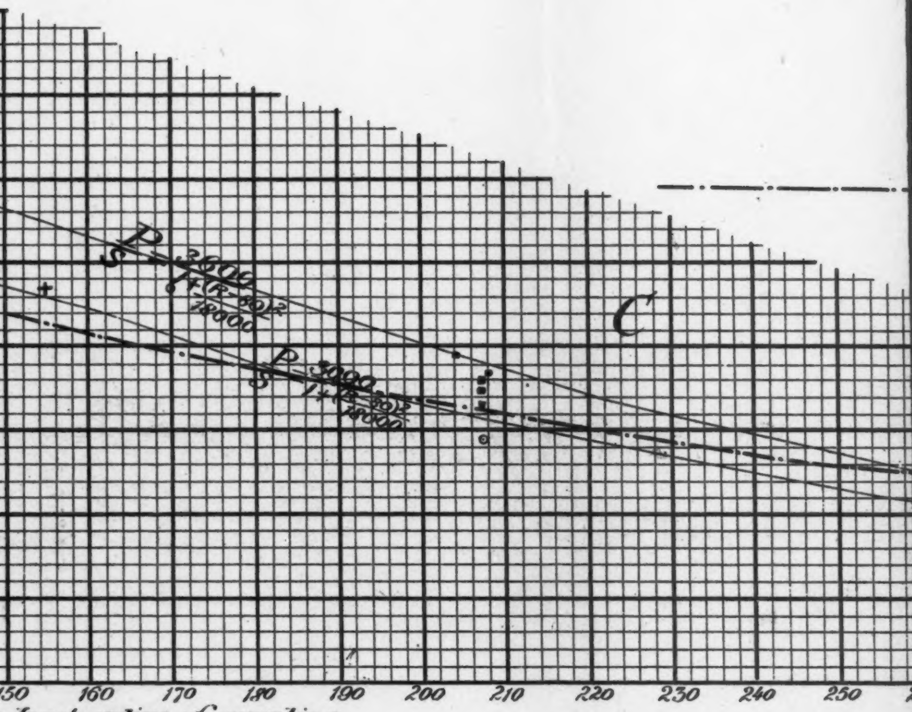


DIAGRAM No. 2

EXPERIMENTS ON SQUARE-

Plotted by ratios of length to least rad

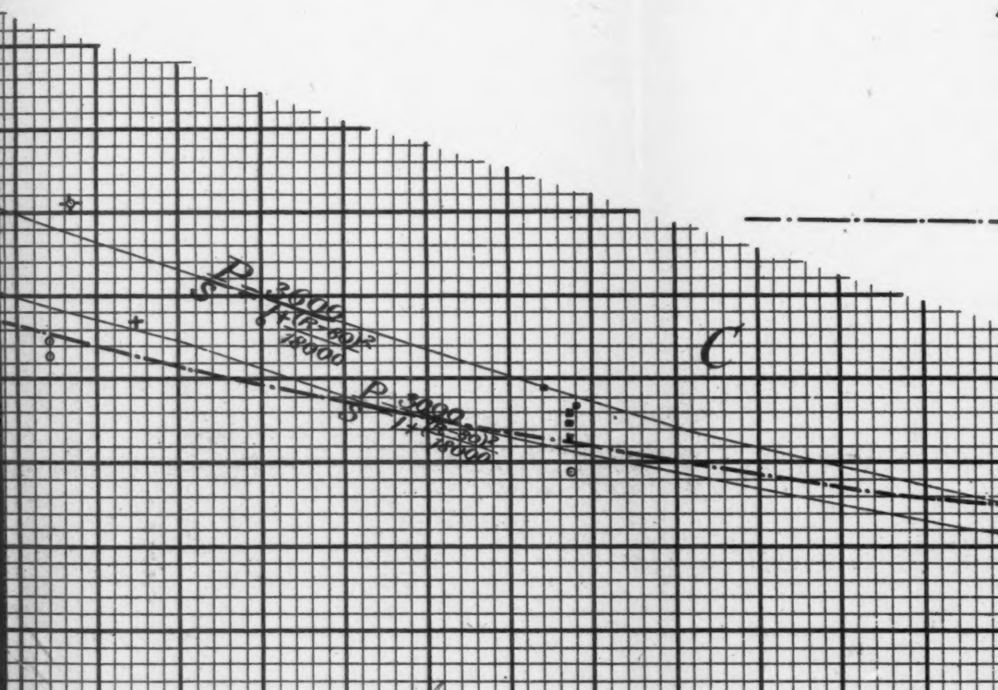


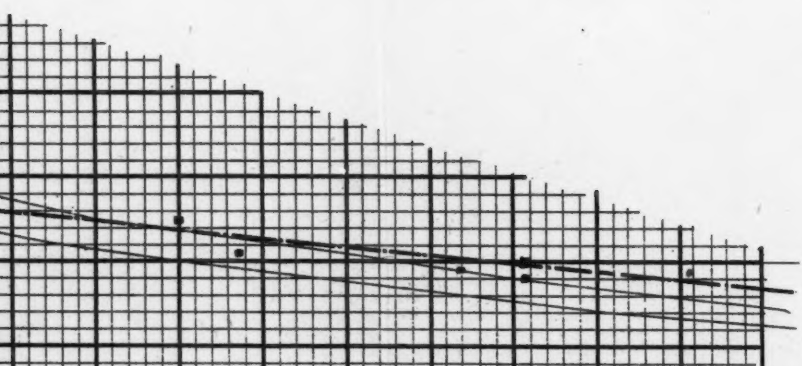
PLATE II.
TRANS. AM. SOC. CIV. ENG'R'S
VOL. XI. Nº CCXXXIII.
COOPER
ON WROUGHT IRON COLUMNS.

2

E-ENDED COLUMNS

radius of gyration $\frac{l}{r} = R$

✧ *Phoenix Columns*
△ *Keystone* "
\+ *Amer Co's* "
□ *Box & Open* "
○ *Wrought iron tubes*
■ *Solid Rectangular Bars.*
—·—·— *Rankine's Formula*



Discussion by D. J. Whittemore, M. A. S. C. E.

I venture to say that if one versed in making tests of building materials should be asked to state how many tests of like kinds should be made, from which to determine a fair average, he would say at least four, probably six. In my experience, tests of compressive strength of all kinds of material used in construction are subject to nearly double the variation found in tensile tests of the same material; therefore, when we have, as in the present case, twenty tests, being only two to each different length of column, and the resulting tests on each length having a variation from the mean, in one instance only, of five per cent.; in three other instances from one and one-half to two per cent., and in the balance the traction of one per cent., we may well express our surprise and admiration at the uniformity attained. These tests were made, however, through the agency of probably the best testing machine of its capacity ever constructed, and the results, in view of the facts above recited, should and will command the attention of every intelligent engineer.

It is true that Gordon's formula does not express the true strength of these columns, as asserted by the authors, neither can it be made to do so by any change in the value of its factors. It is also, I believe, repugnant to our understanding to admit that two formulæ will be required to express strength, when the facts are approximately known. The line of rapid descent in strength appears to have been reached at 15 diameters, and at about the point where the ratio between the thickness of plates and distance apart of rivets equals the diameter of the column into its length. The two tests at this point show uniformity of strength, but it may be remarked that one of these columns buckled by shearing of rivets, and this was the only instance of the kind, and that the elastic limit of the other was much below any of the others recorded. Is it in reason to suppose a column 15 diameters long to be no stronger, or not even as strong, as one of 18 or 24 diameters in length?

Had there been a variation of results in these columns as great as was found in the tests of the columns $10\frac{1}{2}$ diameters long, giving an increased mean value to the same, the results would not appear so anomalous, as they now certainly do. All experiments are immensely suggestive of further tests, and the only conclusive solution that I can think of would be to test at least four more similar specimens of 15

diameters in length. It also occurs to me that the strength value of posts of the size tested might be very materially increased between 9 and 36 diameters in length by riveting the segments together after some such rule as the following, *i. e.*, distance apart of rivets in inches shall not exceed the square root of the length of the column as measured by its diameter.

Undoubtedly the laws governing force and matter can be expressed by formulæ, and as an indication that the case in hand is not an exception, from several formulæ I have devised, I select the following as an expression of the probable ultimate strength of these columns, as indicated by the experiments cited, *i. e.*:

$$S = (1\,200 - H)\,30 + \frac{525\,000}{H^2}$$

$$\text{Where } H = \frac{\text{Length}}{\text{Outside diameter}}$$

Evidently this formula will not apply to columns longer than 45 or shorter than about 5 diameters in length, but it will be observed that covering the tests mentioned by the authors, the average variation from all tests does not exceed one per cent., though in the one instance, that at 15 diameters the variation amounts to four per cent. In the following table I have entered in the column of actual strength the average of the two tests on each length of post, and also the single tests on eleven-inch columns of 9 and 25½ diameters :

$H.$	Actual Strength per Square Inch, by Experiment.	By Formula. $S=(1\,200-H)\,30+\frac{525\,000}{H^2}$
6	50 370	50 400
9	42 180	42 224
$10\frac{1}{2}$	40 728	40 447
15	36 438	37 883
$19\frac{1}{2}$	37 028	36 796
24	36 580	36 191
$25\frac{1}{2}$	36 010	36 043
$28\frac{1}{2}$	36 132	35 791
33	34 965	35 492
$37\frac{1}{2}$	35 155	35 248
42	34 650	35 038

$H=\frac{L}{D}$: as given by the authors of the paper in the column headed, "Ratio of Diameter to Length."

Discussion by Charles E. Emery, M. A. S. C. E.

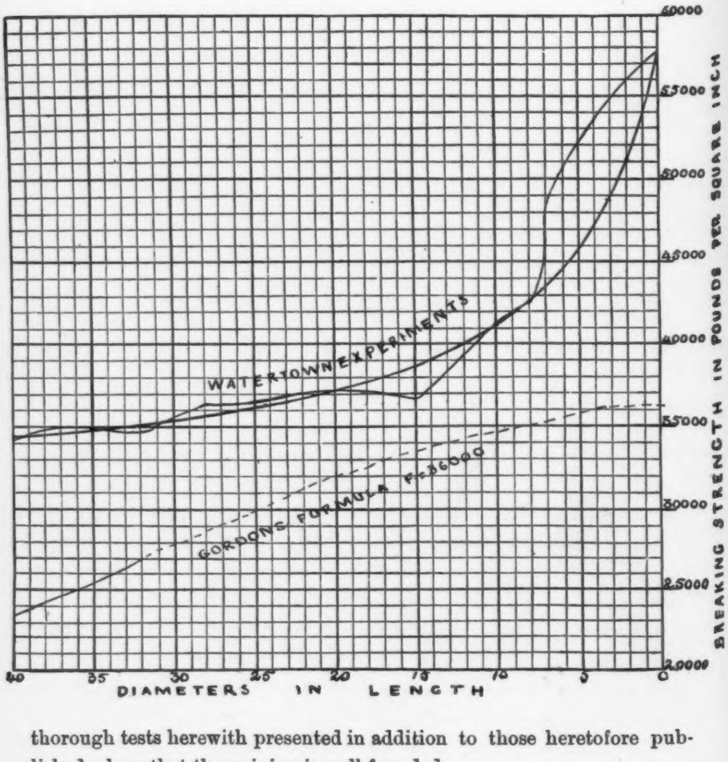
I have taken great interest in this valuable paper, and trust that the example set by this well known firm may be followed by others who have occasion to use the Government Testing Machine, as various experiments, carefully analyzed and discussed here, will be the means of greatly extending the information of engineers on this important subject.

It has probably been the impression of many here present that heretofore each writer, in discussing experiments on this subject, has constructed a formula with an ample factor of safety, to which compilers have added an additional factor of safety to cover exceptional cases, and that this process has been continued until the formula no longer represents the actual facts, but give results unnecessarily safe when good

workmanship and material and symmetrical sections of approved form are provided.

This state of facts has undoubtedly been caused, to a great extent at least, by the differences in result obtained in different experiments, particularly with columns made of cast-iron of different grades, and constructed without special pains to secure uniform thickness and freedom from defect.

Engineers have for a long time felt that the segmental circular columns of Clarke, Reeves & Co., were far more reliable than those upon which the greater part of our literature on the subject is based, and the



thorough tests herewith presented in addition to those heretofore published, show that the opinion is well founded.

The diagram presented with the paper indicates clearly that two formulæ are needed to represent the experimental curve accurately, as has been pointed out previously by many writers on the general subject in relation to other experiments.

In examining these experiments for the purpose of developing a simple formula for my note-book, I find that the whole of the experimental curve can be represented approximately by one formula which will be very accurate within the limits where formulæ are generally used, and not vary seriously for shorter lengths. The curve is a hyperbola, with the equation :

$$y = \frac{355\,063 + 30\,950\,x}{x + 6.175} \qquad (a)$$

in which y represents the breaking load in pounds per square inch, as shown by the ordinates in the diagram ; and x, the number of diameters in length, as shown by the abscissa of diagram.

The curve developed by this equation has been plotted on one of the printed diagrams presented by the writers of the paper, and a copy of the same is herewith reproduced. It will be seen that the curve is very accurate from twenty diameters upward, and corresponds with the experimental curve better in form and position than that given by Gordon's formulæ, as plotted by the writers of the paper. At 15 diameters the results given by the formula are higher than the experimental ones, but for a less number of diameters the experimental results are the higher, making the formula as a whole perfectly safe, as the slight variations will be provided for in the factor of safety.

Discussion by De Volson Wood, M. A. S. C. E.

Engineers are naturally interested in these experiments, not only on account of the information which they will gain in regard to the resisting properties of the Phœnixville column, but also because the results determined are more reliable since the tests were made with the most accurate testing machine the world has ever known. It is a matter of pride that this country possesses the only machine with which the actual stresses, even though they amount to hundreds of tons, may be determined within the fraction of a pound.

In regard to the results of these experiments, it is remarkable that the law of strength appears to change so suddenly with columns of 15

diameters, the value originally indicated by Hodgkinson. For larger columns the law seems to be approximately, but not rigidly, represented by Gordon's formula. The line representing the computations by Gordon's formula seems to drop more rapidly than the line representing the results of the actual experiments, and for this reason Gordon's formula may need a correction in order to represent these experiments more accurately. But I do not understand why the line representing Gordon's formula should be so far below the other. The constants in his formula should be determined by means of these experiments and not from the results of English experiments, since the form of column and the material are both different. Had this been done the curve would have nearly coincided with some portion of the experimental curve. The formula may be written.

$$P \text{ (strength in pounds)} = \frac{B \text{ (constant)} \times A \text{ (area in sq. in.)}}{1+C \text{ (constant)} \times r_2 \left\{\begin{array}{l}\text{square of}\\ \text{the ratio of}\\ \text{length to}\\ \text{radius of}\\ \text{gyration.}\end{array}\right\}}$$

It is sometimes stated that B in this formula is the resistance to ultimate crushing. This would be the case if the law of resistance were true without limit; for when $l = o$, we have $P = B A$. But the law cannot be safely extended much, if any, beyond the limits of the experiments. The true way to find the constants is to substitute known contemporaneous values for P, A, and r, thus forming as many equations as there are experiments, any two of which will give definite values for B and C. If the formula represents the law of strength with sufficient accuracy, the mean values of these constants, as determined by different combinations of the equations above formed, will be the proper values to be used in the formula. If the law changes at 15 diameters, or at any other point, the formula should not be distorted so as to take in the whole range of the experiments. Another formula should then be used for lengths less than 15 diameters. I regret that I have not been able to give the time necessary to enable me to carry out these numerical computations, and to give to the whole subject a more critical study.

I desire to make a remark upon the report of the examiner. I observe that the sets which were observed after the stress was removed are generally recorded, while in a majority of cases the total compressions are omitted. The experimenter has indicated the limit of elasticity in several

cases, but so far as the record shows, these limits are arbitrary. A complete set of all quantities observed should be recorded to enable any reader to judge for himself of the accuracy of the work, and the correctness of the conclusions, as well as to permit of more accurate comparison with other experiments.

Discussion by Charles E. Emery, M. A. S. C. E.

In reply to Professor Wood.—We are all very much interested in Professor Wood's clear discussions of scientific subjects, but I must confess that I cannot agree with him that Gordon's formula even approximately represents the curve shown by the experiments under discussion. The variation is not that simply due to a lower position on the diagram, but within the limits mentioned, Gordon's formula gives a curve concave to the axis of abscissas, while the experimental curve is evidently convex to such axis. This is an important difference, for it is evident that an equation of a straight line would more accurately represent the experimental curve than the curve given by Gordon's formulæ between the limits shown. In fact, the experiments are very well represented by two straight lines with the following simple equations, viz.:

First.—

$$y = 39\,220 - 118\,x \qquad (b)$$

which is very accurate between the limits, 12 to 40 diameters, safe for a less number of diameters, and probably also for a greater number within reasonable limits.

Second.—

$$y = 57\,500 - 1\,642\,x \qquad (c)$$

which is approximately accurate between the limits 0 to 12 diameters, the notation being the same as before.

Discussion by C. L. Strobel, M. A. S. C. E.

The interesting series of tests on Phœnix columns instituted by Messrs. Clarke, Reeves & Co., deserve careful study, as throwing additional light upon an important engineering subject, still much involved in obscurity. I think, however, that their reference to Gordon's formula implies a misapplication of the same, and gives an improper interpretation to Hodgkinson's tests from which the constants in the formula were deduced; and I shall attempt to show that these tests on Phœnix columns do not conflict with the latter, but, on the contrary, serve to strengthen and confirm the conclusions to which they have led.

Of the 22 Phœnix columns tested, 20 are four-segment columns of 8.04 inches diameter from out to out of cylindrical portion. The areas of these 20 columns are practically the same, the average being 12.122 square inches; the diameter is given at 11.5 inches from out to out of flanges. Calculating the thickness of metal from these data, we obtain 0.32 inch for the thickness of the cylindrical portion and 0.31 inch for the mean thickness of the flanges, whence the moment of inertia is found to be = 109.72, and the square of the radius of gyration = 9.05.

The general form of Gordon's formula, applicable to any cross section, is the following:

$$\frac{P}{S} = \frac{f}{1 + a\frac{l^2}{r^2}} \qquad (1)$$

In this formula P represents the total pressure under which the column fails, S its sectional area, and consequently $\frac{P}{S}$ its ultimate strength per square inch; f is a constant dependent upon the form of section and the ultimate strength of the material; a is a constant also dependent upon the form of section and the ultimate strength of the material, but also dependent upon its elasticity, being inversely proportional to the modulus; l = the length of column; r = the radius of gyration of the section. The constants f and a are determined by experiment.

Applying the above formula to a hollow cylinder or tube, and letting d represent the diameter of an equivalent circle whose radius of gyration,

assuming the area concentrated in the circumference, is equal to that of the section of the tube, we have, for this form of cross section,

$$\frac{P}{S}=\frac{f}{1+8a\frac{l^2}{d^2}} \qquad (2)$$

since $r^2=\frac{d^2}{8}$

Again, applying the general formula to a column of solid rectangular section whose least side $= h$, we have,

$$\frac{P}{S}=\frac{f}{1+12a\frac{l^2}{h^2}} \qquad (3)$$

since $r^2=\frac{h^2}{12}$

The general form (1) of the formula has sometimes been called Rankine's, while the special forms (2) and (3) which the former assumes by substituting for r^2 its equivalent in terms of d^2 or h^2, has been designated Gordon's. I have not thought it proper to preserve this distinction, and there is, of course, no difference in the results obtained from either, provided that, in applying the special forms (2) and (3), the proper "equivalent" diameter or least side is first obtained from the equation $d=\sqrt{8.r^2}$ or $h=\sqrt{12.r^2}$.

Our main reliance for the determination of the constants f and a, has been, to the present time, the tests made by Hodgkinson about 35 years ago, preparatory to the building of the Britannia and Conway tubular bridges. Of his tests on wrought iron, those on bars of solid rectangular section are best suited for the determination of the constants, because their range is greatest and the form of section may be best relied upon to give results least affected by accidental irregularities due to shortcomings in the manufacture. To these tests Gordon applied his formula in the form No. 3, and found that if f was made $=36\,000$ and $12\,a=3\,000$, (units being the pound and the inch), the results of the tests would best conform to the values given by the formula. If we may assume, now, that the constants which apply to one form of cross section, will also hold good for other forms, we obtain from (1) the following general formula for square bearing wrought-iron columns :

$$\frac{P}{S}=\frac{36\,000}{1+\frac{1}{36\,000}\frac{l^2}{r^2}} \qquad (4)$$

We shall see, presently, whether the experiments justify this assumption.

It is well known that short columns do not fail by flexure, but by the buckling or bulging of the metal, and that the strength of such columns is measured by the strength of short portions to resist this tendency, and is independent of their length. Gordon's formula assumes that flexure takes place, and that the direct compressive strain (which is uniform in a column which does not deflect from a straight line), is augmented by a bending strain. It is therefore a question to be determined by experiment, when is the ultimate strength of a column independent of its length and when does Gordon's formula apply. Hodgkinson's experiments on rectangular bars shed no light upon this question because tests on bars between 18 and 36 diameters long are lacking, but they indicate that bars 36 diameters long belong to the class of columns which fail by flexure and to which therefore the formula is applicable. His tests on rectangular tubes are more serviceable in this respect, though they are limited in range, none of them exceeding in length 30 times the least breadth, equivalent to 26 diameters if the diameter of the cylindrical tube be adopted as unity. Referring to these tests Stoney draws the following conclusion : "When the length of a rectangular wrought iron tubular pillar does not exceed 30 times its least breadth, it fails by the bulging or buckling of a short portion of the plates, not by flexure of the pillar as a whole, and within this limit the strength of the tube seems nearly independent of its length. It is quite possible that the ratio of length to breadth of wrought iron tubes might be considerably greater than 30 without very materially affecting their strength, but the recorded experiments do not extend sufficiently far to determine this point." It was found that the crushing unit strain was dependent upon the ratio between thickness of plate and breadth of tube, being generally greater the thicker the plate, and the general rule was deduced that the thickness of plate should not be less than one-thirtieth of the breadth of tube or distance between supports. The average ultimate strength of the rectangular tubes which conformed to this rule was 27 000 pounds per square inch.

Besides the tests on bars and tubes of rectangular section, Hodgkinson made a series of 37 tests on cylindrical tubes, ranging to 85 diameters in length. Comparing the results between 14 and 23 diameters, the differences in the ultimate strength are very small, but between 23 and 32

diameters there is much irregularity, and the evidence is not conclusive that within these limits the ultimate strength is independent of the length. If the best results only are considered, this is the case, even a tube 42.9 diameters long showing an ultimate strength but slightly less than the average. I am inclined to think that much of the irregularity in the results is due to imperfections in the manufacture of these tubes, of most of which it is stated that they were united by "soldering or otherwise." Five of them were formed by bending a plate into the form of a cylinder and making a lap joint along which a single line of rivets was driven, and these tubes invariably gave higher results than the others.

Averaging the results of the tests on lengths between 14 and 32 diameters, we obtain 33 100 lbs. as the ultimate strength per square inch of cylindrical tubes, whence it follows that this form of cross-section is much stronger than the rectangular tubular. This being the case for lengths up to 32 diameters, it is also probably true of lengths exceeding 32 diameters, and comprehensive tests on different forms of cross-section would no doubt show that different constants are required in Gordon's formula for each. The formula in the form (4) cannot therefore be considered as giving correct results for all forms of cross-section, though, in the absence of such tests, it has been necessary to use it without limitation. Applying the formula to the cylindrical tubes tested which exceeded 32 diameters in length, the calculated results are found to agree fairly well with the results by experiment, the latter being even generally higher than the former. This appears strange at first glance, since it would then follow that a solid rectangular column is no stronger than a cylindrical tube, although the former has the advantage of holding its material better together, making wrinkling or buckling impossible. But an explanation is easily found in the circumstance that the width of base of the rectangular column is smaller than that of the cylindrical tube in the ratio of 8 to 10, nearly, and that the rectangular column has consequently less fixity of ends than the other. In fact, strictly speaking, columns without discs cannot be considered as "fixed" at the ends, since, being made of elastic material, these will necessarily yield somewhat to a pressure which tends to bend the column ; and when it is considered that a column with round ends is, as regards flexure, the equivalent of a square bearing column of twice its length, the importance of fixity at ends is evident.

In view of the importance and value of Hodgkinson's tests, even to

the present day, inasmuch as they still form our main guide in the proportioning of members subjected to compression, and for the reason that, so far as I know, they have not heretofore been published in convenient form for reference and comparison, I have prepared a tabular statement of his tests on the compressive strength of bars and cylindrical tubes, showing in adjoining columns the ultimate strength per square inch by experiment and by calculation. Gordon's formula in the form (2) was used, in which the length is expressed in terms of the diameter of a tube, in order to facilitate comparison with the Phœnix tests. In every case, however, the proper "equivalent" diameter was calculated from the radius of gyration, so that formula (4) would give exactly the same results.

The tests were made in a lever testing machine, the specimens being placed vertically between square bearing surfaces with ends carefully bedded. The data for the tables were obtained from Edwin Clark's work on the Britannia and Conway Tubular Bridges. Some small discrepancies occur in the thickness given for the tubes, as compared with the outside and inside diameters, but they were unexplained in the original, and could not be corrected.

Reverting now to the tests on Phœnix columns, it is necessary to correct, first, the graphic representation given of the values of $\frac{P}{S}$ obtained by Gordon's formula; by some oversight the values plotted are those for *solid rectangular* sections. Next, it is obvious that the inside diameter of Phœnix columns is not the proper diameter to use in the formula, and that, therefore, the ratio of length to diameter needs revision. For the 8 inch columns r^2 was found $= 9.05$, whence the equivalent diameter is obtained as $\sqrt{8r^2} = 8.51$ inches. Therefore, $\frac{l}{d}$ for the 28 feet columns, experiments 1 and 2, becomes 39.5 instead of 42, and for the 10 feet columns, experiments 13 and 14, 14.1 instead of 15. Comparing experiments 1 to 14, inclusive, with each other, the slight variation in the ultimate strength per square inch is very striking. The average is found to be 35 800 lbs., and the greatest variation is only 1 400 lbs. (for experiment No. 2), while the range is from 14.1 to 39.5 diameters. Of the Phœnix experiments it may therefore be said that they furnish better evidence than Hodgkinson's own tests for the conclusion to which these have led, viz., that the ultimate strength of tubular columns not over 26

HODGKINSON'S EXPERIMENTS ON THE COMPRESSIVE STRENGTH OF CYLINDRICAL TUBES OF WROUGHT IRON.

No.	Length of Tube.	Outside Diameter.	Inside Diameter.	Thickness of Metal.	Area of Section.	Ratio of Equivalent Diameter to Thickness of Metal.	Equivalent Diameter.	Ratio of Length to Equivalent Diameter.	Ultimate Strength per Square Inch. *Actual.*	Ultimate Strength per Square Inch by Gordon's Formula. *Calculated.*	Kind of Tube.	REMARKS.
	Inches.	Inches.	Inches.	Inch.	Sq. In.		Inches.		Lbs.	Lbs.		
1	30	6.125	....	.098	1.799	63	6.028	5	41 360	....	Riveted.	This tube was cut from No. 4.
2	28.3	4.026	....	¼ nearly.	1.88 nearly	15	3.78 nearly.	7.5	48 000 nearly.	....	Gaspipe.	This tube was cut from No. 21, 22 or 23.
3	28.3	4	....	"	"	15	3.76 nearly.	7.5	47 400 nearly.	....	"	This tube was cut from No. 21, 22 or 23.
4	60	6.175	....	.101	1.799	60	6.075	9.9	38 360	....	Riveted.	This tube was cut from No. 18.
5	28.25	3	2.712	·153	1.414	19	2.860	9.9	37 390	....	Gaspipe.	
6	30	2.49	2.275	....	.8045	22	2.385	12.6	36 500	....	"	
7	29.1	2.373	1.911	.231	1.554	9	2.154	13.5	36 900	....	"	Gave way by flexure.
8	29.1	2.343	1.939	.202	1.3587	11	2.150	13.6	39 580	....	"	" "
9	29.9	2.383	1.891	.246	1.651	9	2.151	13.9	33 110	....	"	" "
10	29.9	2.343	1.923	.210	1.407	10	2.143	14	38 220	....	"	" "
11	30	2.34	1.91	....	1.4353	10	2.136	14	36 640	....	"	
12	30	2.335	....	....	1.4353	10	2.131	14.1	35 390	....	"	
13	90	6.366	....	.1298	2.547	48	6.237	14.4	41 660	....	Riveted.	This tube was cut from No. 17.
14	60	3.995	....	.2455	2.895	15	3.760	16	30 020	....	Gaspipe.	This tube was cut from No. 21, 22 or 23.
15	60	3.995	....	.241	2.848	16	3.762	16	34 450	....	"	
16	30	1.964	1.755	....	.6104	17	1.862	16.1	36 980	....	"	Gave way by flexure.
17	120	6.366	....	.1298	.2547	48	6.237	19 2	35 890	....	Riveted.	Unsound spot patched.
18	120	6.187	....	.0939	1.799	65	6.094	19.7	33 390	....	"	
19	30	1.495	1.292	....	.4443	14	1.397	21.5	34 220	....	Gaspipe.	
20	90	4.052	3.79	....	1.613	30	3.923	22.9	33 340	....	"	
21	89.3	4	3.504	.2425	2.873	16	3.760	23.7	27 820	....	"	
22	89.3	4	3.505	.25	2.897	15	3.761	23.7	26 500	....	"	
23	89.3	4	3.511	.2435	2.879	15	3.764	23.7	26 050	....	"	
24	60	2.49	2.275	....	2.385	22	2.385	25.2	35 100	....	"	
25	60	2.35	1.91	....	1.4721	10	2.141	28	29 330	....	"	Gave way by flexure.
26	60	2.335	1.925	.205 nearly.	1.3718	10	2.139	28.1	30 010	....	"	Ditto. Tube not quite straight.
27	119.25	4.05	3.772	....	1.7078	28	3.913	30.5	27 650	....	"	
28	119.25	4.06	3.75	.150	1.9015	26	3.908	30.5	26 240	....	"	
29	90	3.035	2.717	.168	1.414	17	2.880	31.2	29 790	....	"	
30	60	1.964	1.755	....	.6104	18	1.862	32.2	33 300	....	"	Gave way by flexure.
31	119.25	2.995	2 693	....	1.349	19	2.848	41.9	27 690	25 900	"	
32	60	1.495	1.292	....	.4443	14	1.397	43	31 190	25 500	"	
33	119	2.49	2.275	....	.8045	22	2.385	49.9	29 780	23 200	"	Gave way by flexure.
34	120	2.34	1.91	.215	1.4353	10	2.136	56.2	22 180	21 200	"	
35	120	2.35	1.865	....	1.605	9	2.121	56.6	21 500	21 100	"	Gave way by flexure.
36	119	1.964	1.755	....	.6104	18	1.862	63.9	23 190	18 900	"	" "
37	119	1.495	1.292	....	.4443	14	1.397	85.2	14 670	13 800	"	

HODGKINSON'S EXPERIMENTS ON THE COMPRESSIVE STRENGTH OF RECTANGULAR BARS OF WROUGHT IRON USED FOR THE DETERMINATION OF THE CONSTANTS IN GORDONS FORMULA.

HODGKINSON'S EXPERIMENTS ON THE COMPRESSIVE STRENGTH OF RECTANGULAR BARS OF WROUGHT IRON USED FOR THE DETERMINATION OF THE CONSTANTS IN GORDONS FORMULA.

No.	Length of Specimen.	Lateral Dimensions.	Area of Section.	Equivalent Diameter.	Ratio of Length to Equivalent Diameter.	Ultimate Strength per Sq. In. *Actual.*	Ultimate Strength by Gordon's Formula. *Calculated.*	REMARKS.
	Inches.	Inches.	Sq. In.	Inch.		Lbs.	Lbs.	
1	3.75	1.023 x 1.023	1.0465	.835	4.5	52 750	....	Weight sustained without fracture.
2	7.5	" "	"	"	9	48 680	....	Mean of two experiments.
3	15	" "	"	"	18	34 550	....	" "
4	30	1.0235 x 1.0235	1.0475	.836	35.9	25 330	28 000	" "
5	30	.996 x 3.	2.988	.813	36.9	29 650	27 600	" "
6	30	.763 x 3.01	2.297	.623	48.2	27 770	23 700	" "
7	60	1.024 x 1.024	1.0486	.836	71.8	17 270	17 000	" "
8	90	1.53 x 3.	4.59	1.249	72.1	19 990	16 700	One experiment.
9	30	.5026 x 2.9867	1.5011	.410	73.2	16 850	16 400	Mean of three experiments.
10	60	.995 x 3.01	2.995	.813	73.8	18 070	16 300	" two "
11	60	.996 x 5.84	5.8166	.813	73.8	17 700	16 300	One experiment.
12	60	.767 x 3.01	2.309	.626	95.8	12 970	11 800	Mean of two experiments.
13	120	1.51 x 3	4.53	1.233	97.3	10 170	11 600	One experiment.
14	90	1.024 x 1.025	1.0496	.836	107.7	9 750	10 100	Mean of two experiments.
15	90	.9955 x 3.005	2.9915	.813	110.7	9 910	9 400	" "
16	90	.995 x 5.86	5.8307	.813	110.7	9 280	9 400	One experiment.
17	60	.507 x 2.98	1.511	.414	144.9	5 600	6 200	Mean of three experiments.
18	120	.995 x 2.99	2.975	.813	147.6	4 280	6 200	" two "
19	120	.766 x 3.01	2.306	.626	191.7	3 380	3 900	" " "
20	90	.5023 x 2.983	1.4983	.410	219.5	2 410	3 100	" three "
21	120	.503 x 2.98	1.499	.411	291.9	820	1 800	Best of two "

The "equivalent" diameter, or the diameter of a hollow cylinder of equal area and radius of gyration with that of the rectangular section, is equal to 0.8165 times the least side of the rectangulr section.

diameters long is independent of their length; and while these tests made it appear probable that this limit might be extended, the Phœnix tests furnish proof that lengths of even 39.5 diameters should be included. All of the Phœnix columns tested belong, in this light, to the class which does not fail by the flexure of the column as a whole, and to which Gordon's formula does not apply. They are, therefore, not available for the purpose of determining the constants in the formula, and no comparison with the results given by the formula is proper.

The relative strength of Phœnix columns within the limits mentioned, compared with Hodgkinson's cylindrical and rectangular tubes, would be as 35 800 : 33 100 : 27 000. We have no means of knowing whether Phœnix iron of the present day is better or less qualified to resist compressive strain than the iron used in Hodgkinson's tubes 35 years ago, but it would certainly appear reasonable to assume that a part, if not all, of the gain of 2 700 lbs. over the cylindrical tubes is to be attributed to the better form of cross-section of the Phœnix column. The flanges strengthen the cylindrical portion of the column, resisting deformation in a diametrical direction, while any tendency they may have to deflect sideways, in which direction they are weakest, will be resisted by the cylindrical portion in the direction in which it is strongest. Some benefit will also accrue to the Phœnix columns, probably, through their larger base, the projecting flanges acting somewhat in the manner of discs.

So far as the tests on Phœnix columns of less than 14 diameters length are concerned, the results do not differ materially from those obtained by Hodgkinson; like them, they show an irregular and sometimes considerable increase of strength; and the very short lengths simply illustrate the well known fact that if a ductile material is held in place so that it cannot readily get away, it will bear a largely increased compressive strain.

The results of the Phœnix tests are not directly applicable to the posts of bridge trusses, because these are not in the condition of the columns in the testing machine. Where Phœnix columns are used for vertical posts in bridge trusses, the lower ends, generally through the interposition of a foot casting, are pin-bearing, a condition intermediate between a square and a rounded (spherical) bearing, while the upper ends of the columns abut against the underside of a cast joint box, also not the equivalent of the bearing surfaces of the testing machine as regards fixity laterally. It would seem probable, however, that columns

which are pin-bearing at both ends have a uniform ultimate strength irrespective of their length to 23 diameters, and that the ultimate strength for Phœnix columns will average 35 800 lbs. within this limit.

The first tests on columns under conditions corresponding to those which prevail in bridges of the American type, viz,, pin-bearing at both ends, were made by Mr. Bouscaren for the Cincinnati Southern Railway, and published in the report of its then Chief-Engineer, Mr. T. D. Lovett, in 1875. They have been of great practical service as furnishing the means of at least approximately determining the constants in Gordon's formula for columns of this class, but additional experiments are much needed. The tests on square-bearing columns made by Mr. Bouscaren and published with the above in the December number of the Transactions of 1880, are mostly on lengths under 30 diameters and cannot be used for the determination of the constants for the different classes of columns experimented with, but they furnish additional proof of the correctness of the conclusion that tubular columns not over, say, 39 diameters long, have a uniform ultimate strength per square inch independent of the length. The following exhibit will make this very clear, in which the numbers of the experiments correspond to the numbers in the tables accompanying Mr. Bouscaren's paper, and the "equivalent" diameters again refer to diameters of cylindrical tubes, and are calculated from the radius of gyration.

Columns 38, 41 and 42, the iron for which was rolled by the same mill at about the same time, and which were built at the same works and tested in the same machine, show, it may be said, exactly the same ultimate strength per square inch for lengths of 21.4, 25.6 and 31.8 diameters. From these tests we may infer that the strength of columns made of two channels latticed, is at least 32 300 lbs. per square inch, as compared with the averages given above for the Phœnix and the Hodgkinson columns.

Experiment No. 37 requires explanation. Mr. Bouscaren attributes the low result which this column gave to the small thickness of webs of channels, which was a little less than $\frac{1}{30}$th of the distance between inside of flanges. But the webs in column No. 38 are no thicker, this column being, in fact, a duplicate of No. 37, except that the flanges of the channels are somewhat heavier, yet No. 38 stood fully as well as columns 41 and 42, which have heavy webs. Though these channels are all rolled by the same mill, there is a marked difference in the mode

SQUARE-BEARING COLUMNS—G. BOUSCAREN'S EXPERIMENTS.

No.	Length.	Equivalent Diameter.	Ratio of Length to Equivalent Diameter.	Ultimate Strength per Square Inch.	Kind of Column and Remarks.
	Feet.	Inches.		Pounds.	
23	24.	9.64	29.9	33 200	Closed column. 2 8-inch channels and 2 10-inch plates. Iron from different mill than for other tests.
32	27.	9.34	34.7	30 200	Closed column like above, but plates 10.5 inches.
22	26.	8.65	36.1	30 000	Closed column like No. 23, but channels 7 inches.
37	27.47	12.64	26.1	29 600	Open column, 2 12-inch "plate" channels latticed 5-16th. web.
38	23.01	12.87	21.5	32 300	2 12-inch channels latticed, 5-16th. web.
41	27.5	12.90	25.6	32 400	2 12-inch channels latticed, heavier web.
42	27.5	10.37	31.8	32 300	2 10-inch channels latticed.
40	1.61	2.366	8.2	35 700	1 12-inch channel, 5-16th. web.
39	2.00	"	10.1	35 400	Same as above.

of manufacture. Nos. 38, 41 and 42 were rolled from a regular channel pile in the usual manner, while No. 37 (Union Iron Mills, Shape No. 26), is made by bending a plate to the form of a channel in the last two passes through the rolls. The channel produced by this process is inferior in strength and shape to the others, and this is, I think, the true explanation of the low strength developed by the column. Hodgkinson's rectangular tubes were composed of plates similarly bent, no channels or angle irons as now made by the mills entering into their construction, and they also gave lower results than appears attributable to their form only.

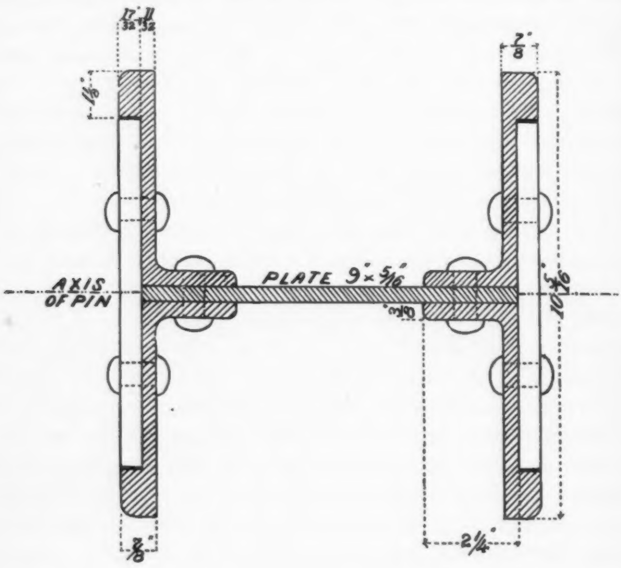
Mr. Bouscaren, in his paper above alluded to, infers from the results of his experiments, that "iron of the highest modulus does not necessarily make the strongest column," presupposing, of course, the other conditions to remain the same; and in the comparisons I have made, differences in the quality of the iron as expressed by the modulus of elasticity and the ultimate compressive strength of the material, were not

alluded to. The latter is an indeterminable value for a ductile material like wrought-iron, and the former is seldom measured with sufficient accuracy to be of service. But both modulus and strength are not very variable quantities for wrought-iron, and I do not think that, had they been properly considered for the above tests, the results obtained would have been materially modified. Mr. Bouscaren's inference will not harmonize with the theoretic basis of Gordon's formula, and is supported only by the fact that the closed rectangular columns tested by him, while giving a lower ultimate strength, showed a higher modulus than the Phœnix columns. But this result, as we have seen, was to be expected as the effect of the different forms of cross-section irrespective of the modulus, and the conclusion, therefore, does not seem to be well founded. If his experiments on Phœnix columns be alone considered, it is true that the highest modulus corresponds to the lowest ultimate strength, but, per contra, nearly the same modulus (difference only 300-000) in another test gave the highest ultimate strength.

In accordance with theory, the greater the modulus and consequently the stiffer the column, the smaller will be the bending moment which it will have to resist, and the greater its ultimate strength. Future experiments may show that the variations in the modulus are of sufficient influence upon the strength of long columns as to require special consideration even for wrought iron, but there can be no doubt that the range of these variations is so great for steel, that no compression formula for this material can be considered complete which does not either limit its application to steel of a certain modulus, as well as ultimate strength, or in which the constant in the denominator is not a function of both. Very mild steel has a modulus probably smaller than that of ordinary wrought-iron, and, although its ultimate strength is greater, it is questionable whether it will sustain more or as much as the latter in the form of long columns. Again, if the modulus is the same, while the ultimate strength of the steel is to that of the iron, say, as 8 to 5, the strength of the steel column will bear some lower ratio to that of the iron column, because the higher pressure which the former sustains will produce a greater deflection from the straight line, and the bending moment will therefore be increased not only in proportion to the greater force, but also in proportion to the greater leverage. While the saving in material for tension members is directly proportional to the greater strength

of the material employed, the saving for long compression members will be a different and smaller ratio.

The following two tests, not heretofore published, are presented in illustration of this principle, showing the amount to which the increased strength of the material is counteracted by the absence of a proportionate increase in the modulus. The tests were made on two columns, one of steel and the other of iron, both of 19 feet length between centres of pins, and 20 feet long out to out, and of the following sections :



$r^2 = 7.6$

Equivalent diameter $= 7.8$ inches.

$\frac{l}{d} = 29.2$

Both the steel and the iron were rolled and the columns built by Andrew Kloman, deceased, at the Superior Mills in Pittsburgh. They were tested horizontally in the Keystone Bridge Co.'s testing machine, in the position corresponding with sketch, counterweighted at the centre by a load equal to one-half the weight of the column. The steel was made by the Besse-

mer process, under the Hay patent, and was furnished under specifications requiring an ultimate tensile strength of 80 000 pounds per square inch. The column was one of a lot built for a steel bridge in 1878. Unfortunately, measurements of the contraction under strain were not taken, and the modulus cannot, therefore, be given. The sectional area was 14.8 square inches for the steel, and 13.2 square inches for the iron column, obtained from the weight. The dimensions given in sketch correspond more nearly to the area of the steel than to that of the iron column. The pins were $3\frac{7}{8}$ inches in diameter. Both columns failed by deflecting downward. The ultimate strength attained in the case of the steel column was 30 900 pounds per square inch ; and in the case of the iron column, 24 800 pounds per square inch. While, therefore, the ultimate strength of the steel may be said to have been 1.60 times greater than that of the iron, the strength of the steel column was only 1.25 times greater than that of the iron column.

In Mr. Howard's report on the Phœnix experiments, he states that the Phœnix columns show a superior sustaining power after taking a deflection of several inches, and therein differ materially from lattice columns, which, after deflecting slightly, suddenly give way by tearing out the lattice bars. I understand that this behavior was observed on one lot of lattice columns only, and Mr. Howard informs me that similar columns tested later, and obtained from a different manufacturer, did not fail in this manner. Of the lattice columns tested by Mr. Bouscaren, and of other columns tested at the works of the Keystone Bridge Co., none have failed at the rivets of the lattice bars, so that the behavior described cannot be said to be general for lattice columns, but must be confined to the particular lot of columns first tested, and is to be attributed, no doubt, to defective *hand*-riveting.

In the foregoing remarks the ultimate compressive strength of the material has been referred to, though it was stated that its value cannot be definitely ascertained. In consequence of this difficulty it has been customary to make tension tests on small specimens for iron and steel intended for compression in the same manner as tests are made on tension material, on the assumption that material giving a high ultimate strength in tension, would also give a high ultimate strength in compression.

Practically, the testing of material for quality must be done on small specimens cut from the large pieces. While it would not be impossible

to ascertain the modulus in compression for these, it would be attended with much difficulty, and could not be done with our present testing machines. In place of the compression modulus it will, therefore, be advisable, also, to substitute the tension modulus, so that, generally, it may be said that it is necessary to judge of the compression qualities of the material by its behavior under tension. That being the case, I consider it very important that experiments on the compressive strength of compression members be always supplemented by tension tests on small specimens of the same material. This has not heretofore been done, and we therefore know nothing concerning the strength and other qualities of the iron used in former tests. It is particularly to be hoped that no experiments on steel columns will be made in future which are not accompanied by tension tests on small specimens and that special attention be given the modulus.

Discussion by A. S. C. Wurtele, M. A. S. C. E.

On advance sheet sent me I wish to make the following remarks :

The tables of experiments on Phœnix columns recorded in paper presented by Clarke, Reeves & Co., appear to be definite, as far as regards the strength of one particular column of about 12 inch section, but are unfortunately not sufficient to generalize on; and it is to be regretted, that the ends were not more carefully prepared and also that no tests were made showing the quality of the iron used in the columns.

Without questioning the value of the Phœnix column, I would ask on what evidence the statement is made, that a lattice column gives away suddenly after a slight deflection ; which statement appears to me to be entirely irrelevant to the tenor of the paper.

Rankine shows the theoretic deduction of Gordon's formula as an approximation, but there is nothing to warrant the continued use of factors, known to be too small for good American iron, as displayed and spread on specifications for iron bridges.

The thanks of the profession are due to Clarke, Reeves & Co. for giving us the full tables in the paper ; which would be greatly increased in value if we could have similar tables for different sections and shapes.

DISCUSSION BY WILLIAM H. BURR, ASSOC. A. S. C. E.

While laboring under permanent ineligibility to membership by a somewhat anomalous rule of this Society, it is with the greatest hesitation that I venture to join in the discussion of this paper, which touches a subject at once interesting and important. The character of the subject must therefore furnish the excuse for my presumption.

At first sight (and the impression does not wear away by continued examination) it is a matter of no little conjecture to discover why Messrs. Clarke, Reeves & Co. should make a comparison of the Watertown experiments with the results of the application of the old form of Gordon's formula, which was designed and only intended for solid rectangular columns of wrought-iron of confessedly less ultimate compressive resistance than that of the present American production. It scarcely seems possible that a deliberate neglect of the influence of the form of cross-section and quality of the material was intended, yet such an inference would be legitimate.

It is not difficult to find a form of Gordon's formula (some may prefer to call it Tredgolds), which shall fit the whole range of experiments without great discordance, while with two forms of formula, as will presently be shown, very accurate results may be obtained.

The form of Gordon's formula, to which reference was just made, is the following:

$$p = \frac{42\,000\left(1 + log\,(1 + \frac{r}{l})\right)}{1 + \frac{1}{50\,000}\,\frac{l^2}{r^2}}\; ; \ldots\ldots\ldots\ldots(1)$$

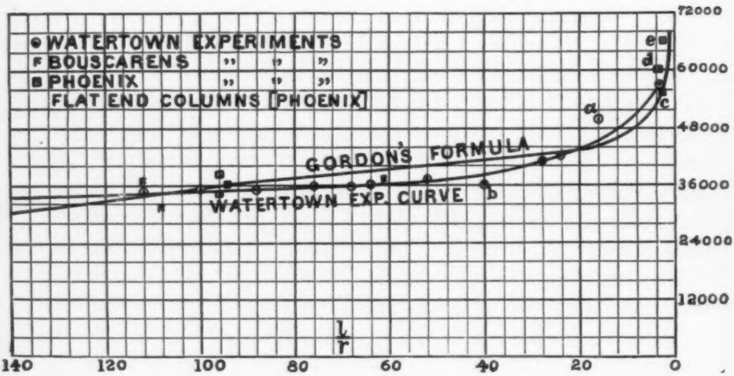
In which r = radius of gyration of cross-section in inches.

l = length of column in inches,

p = ultimate compressive resistance in pounds per square inches.

log = Naperian or hyperbolic logarithm.

In the following diagram I have represented graphically the results of the application of Eq. (1), and the experimental results given by the United States testing machine at Watertown.



The curve indicated by "*Gordon's Formula*" was constructed from Eq. (1) ; the other one was drawn so as to best fit the Watertown experiments.

As a matter of interest, I have also plotted the results of the experiments by Mr. Bouscaren, as well as those of some experiments made by the Phœnix Iron Co. at different times. The different classes of experiments are clearly indicated in the diagram.

As is shown, the horizontal scale represents the quotients $\frac{l}{r}$, while the vertical scale represents pounds.

It is thus seen that the greatest discrepancy between Eq. (1) and the experiments is not over 10 per cent. This, perhaps, is sufficiently close for all ordinary purposes, yet more accurate formulæ are desirable, and may be obtained.

It is interesting and important to observe that each experimental value in the diagram (which, for the Watertown experiments, is a mean of two, belonging to columns of the same length), lies on, or exceedingly close to the curve, with the exception of those shown at *a* and *b*. *a* corresponds to a mean of Nos. 17 and 18 in the table (of the paper under discussion), and is abnormally high. *b* shows the mean of Nos. 13 and 14, and is abnormally low.

It may be remarked that the experimental curve is nearly a straight line from a point just above *b* to the extreme left of the diagram. For

that portion of the curve it will be found that the following formula applies very closely:

$$p' = 39\,640 - 46\,\frac{l}{r}\,; \quad \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

p' representing the ultimate resistance per square inch. The results of the application of this formula are given in the column headed p' in the following table. The table, in connection with the diagram, shows that this formula may be used with accuracy for values of $l \div r$, lying between 30 and 140, and further experiments may possibly show that it is applicable above the latter limit.

For values of $l \div r$ less than 30, the following formula will be found to give results approximating very closely to the experimental curve:

$$p'' = 64\,700 - 4\,600\sqrt{\frac{l}{r}}\,; \quad \ldots\ldots\ldots\ldots\ldots(3)$$

The results of the application of this formula are given in the column headed p''.

The extreme simplicity of Eqs. (2) and (3) makes it a matter of great interest and importance to determine, by other experiments covering extended ranges of $l \div r$, whether these forms with different constants may not apply to shapes other than that of the Phœnix columns.

The results of the application of Eqs. (2) and (3) to Bouscaren's and the Phœnix experiments are not given, but the diagram shows that they would be satisfactory.

It is a question whether the degree of distortion which accompanied the extremely high result of 65 867 pounds per square inch (shown at e in the diagram), was not considerably greater than that which would characterize the condition of "failure" in actual structure. This important point cannot receive too much attention in connection with short column tests, when the relative distortion, in the condition of "failure," is far greater than in long columns.

When $l \div r$ becomes larger than 60, the term $log\ (1 + \frac{r}{l})$, in Eq. (1) may be omitted.

No.	r^2.	$l \div r$.	*Exp.*	*p*.	p'.	p''.
1	8.94	112	35 150	33 870	34 488	
2	"	"	34 150	"	"	
3	"	100	35 270	35 350	35 040	
4	"	"	35 040	"	"	
5	"	88	35 570	36 780	35 592	
6	"	"	34 360	"	"	
7	"	76	35 365	38 100	36 144	
8	"	"	36 900	"	"	
9	"	64	36 580	39 400	36 696	
10	"	"	36 580	"	"	
11	"	52	36 857	40 600	37 248	
12	"	"	37 200	"	"	
13	"	40	36 480	41 700	37 800	
14	"	"	36 397	"	"	
15	"	28	38 157	42 800	38 352	40 360
16	"	"	43 300	"	"	"
17	"	16	49 500	44 300	....	46 300
18	"	"	51 240	"	....	"
19	"	2.7	57 130	55 230	....	57 140
20	"	"	57 300	"	....	"
21	19.37	68.8	36 010	38 900	36 666	
22	"	24	42 180	43 200	....	42 160

DISCUSSION BY MANSFIELD MERRIMAN, J. M. A. S. C. E.

Gordon's Formula has, on account of its theoretical basis, obtained a wide acceptance as an expression of the law governing the strength of columns longer than about twelve or fifteen diameters. If A be the cross-section of the column, P the load that breaks it, l its length, and d its least diameter, the crushing unit strength is according to this formula:

$$\frac{P}{A} = \frac{S}{1 + T\frac{l^2}{d^2}}$$

where the constants S and T are to be determined rom experiments and depend upon the material, form of cross-section and arrangement of ends of the column. Two experiments furnish two equations from which the values of S and T may be found. When more experiments than two are given, the values of S and T are to be deduced by the method of least squares. For experiments Nos. 1 to 14 of the paper of Messrs. Clarke, Reeves & Co., the most probable values of S and T are thus found to be 37 200 pounds and $\frac{1}{22\,530}$, so that for these experiments the formula is

$$\frac{P}{A} = \frac{37\,200}{1 + \frac{1}{22\,530}\frac{l^2}{d^2}} \qquad \ldots\ldots\ldots\ldots (1)$$

provided P be taken in pounds, A in square inches, and l and d both in the same linear unit.

The form of Gordon's formula which uses r the least radius of gyration in the place of d the least diameter, is preferred by many on account of its wider theoretical basis. For this case experiments Nos. 1 to 14 give

$$\frac{P}{A} = \frac{37\,200}{1 + \frac{1}{158\,500}\frac{l^2}{r^2}} \qquad \ldots\ldots\ldots\ldots (2)$$

as the most probable formula.

An inspection of the results of these experiments indicates that, for columns longer than about fifteen diameters, the decrease in the ultimate unit strength is approximately proportional to the length of the column expressed in diameters. That is to say

$$\frac{P}{A} = S - T\frac{l}{d}$$

in which S and T are to be found from experiments for each kind of column (and are not at all the same as the S and T in Gordon's formula). The application of the method of least squares gives 38 240 and 83.8 pounds for the values of S and T from experiments Nos. 1 to 14, so that

$$\frac{P}{A} = 38\,240 - 83.8\frac{l}{d} \cdots\cdots\cdots\cdots (3)$$

is the most probable expression of this law.

The following table gives the values of the ultimate unit strain $\frac{P}{A}$ calculated from formulas (1), (2) and (3) for eleven different values of $\frac{l}{d}$ or $\frac{l}{r}$, these eleven values being those corresponding to the fifteen

Nos.	$\frac{l}{d}$	$\frac{l}{r}$	Calc. (1)	Calc. (2)	Calc. (3)
13 and 14	15	40	36 840	36 840	36 980
11 and 12	19.5	52	36 590	36 590	36 610
6*	22.4	62	36 400	36 330	36 300
9 and 10	24	66	36 290	36 290	36 230
21	25.5	68	36 160	36 150	36 100
7 and 8	28.5	76	35 920	35 920	35 850
5 and 6	33	88	35 490	35 490	35 480
3 and 4	37.5	100	35 020	35 020	35 100
10*	39.5	108	34 770	34 640	34 900
28* and 29*	40.7	112	34 660	34 500	34 830
1 and 2	42	112	34 500	34 500	34 720

long Phœnix columns, Nos. 1 to 14 and No. 21, experimented upon by Messrs. Clarke, Reeves & Co., and the four Phœnix columns tested by Mr. Bouscaren in 1875, and reported in the Transactions of this Society for December, 1880; these four are marked with an *. For formulas (1) and (3) the values of $\frac{l}{d}$ are used, and for formula (2) those of $\frac{l}{r}$. The corresponding values of $\frac{P}{A}$ for the three cases are given in the columns marked Calc. (1), Calc. (2) and Calc. (3). These exhibit a very close agreement.

In the following table is given a comparison of the observed values of the ultimate unit strength with those above calculated from the three formulas. The third column contains the values of $\frac{P}{A}$ found by ex-

No.	$\frac{l}{d}$	Observed.	Diff. (1).	Diff. (2).	Diff. (3).
13	15	36 480	— 360	— 360	— 500
14	15	36 400	— 440	— 440	— 580
11	19.5	36 860	+ 270	+ 270	+ 250
12	19.5	37 200	+ 610	+ 610	+ 590
6*	22.4	37 500	+ 1 100	+ 1 170	+ 1 200
9	24	36 580	+ 290	+ 290	+ 350
10	24	36 580	+ 290	+ 290	+ 350
21	25.5	36 010	— 150	— 140	— 90
7	28.5	35 360	— 560	— 560	— 490
8	28.5	36 900	+ 980	+ 980	+ 1 050
5	33	35 570	+ 80	+ 80	+ 90
6	33	34 360	— 1 130	— 1 130	— 1 120
3	37.5	35 270	+ 250	+ 250	+ 170
4	37.5	35 040	+ 20	+ 20	— 60
10*	39.9	31 000	— 3 770	— 3 640	— 3 900
28*	40.7	34 800	+ 140	+ 300	— 30
29*	40.7	36 600	+ 1 940	+ 2 110	+ 1 770
1	42	35 150	+ 650	+ 650	+ 430
2	42	34 150	— 350	— 350	— 570

periment, and the fourth, fifth and sixth contain the differences between these observed values and those given in the first table. It will be seen that formula (2), which is theoretically the most satisfactory, exhibits no closer agreement than (1), and that (3), which is entirely of an empirical nature, gives nearly as close an agreement as any of the others—in fact, if experiment No. 10* be omitted, the precision of (3) is superior to (2). It hence appears that the simple formula

$$\frac{P}{A} = 38\,240 - 83.8\,\frac{l}{d}$$

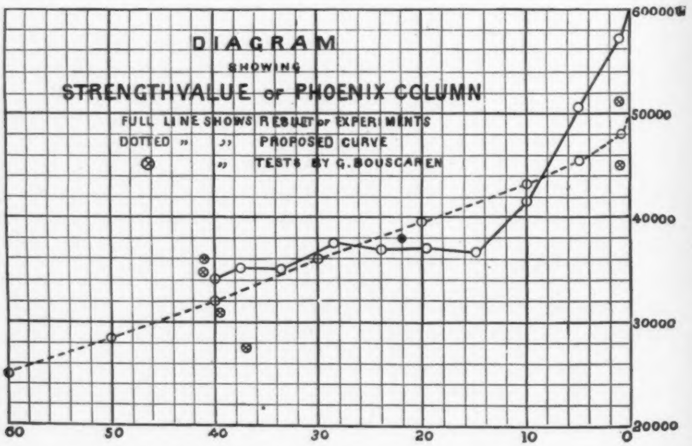
may be said to represent very satisfactorily the ultimate strength of the

Phœnix columns with flat ends and greater in length than about twelve diameters, which have thus far been tested.

Discussion by C. L. Gates, J. M. A. S. C. E.

From a graphical representation of strength of columns experimentally determined, as per foregoing paper, an intelligent analytical reproduction of strength value for different lengths would at first seem out of the question. Yet, from the fact that experiments Nos. 13 and 14, showing unusual low limit of elasticity and, therefore, poorer quality of material, may be neglected and experiments recorded by G. Bouscaren, see Transactions, December 1880, for one, twenty-two, thirty-seven, and forty diameters, change said average strength value somewhat, lead me to propose for the consideration of the Society, for crippling strength of flat Phœnix column the following formula:

$$S = \frac{50\,000 - 2\,000\sqrt{H}}{1 + \frac{H^2}{10\,000}}$$



which differs from Gordon's formula only in that the denominator instead of being a constant, represents the formula of a parabola whose apex is at 50 000 lbs. the assumed absolute crushing strength of wrought-iron, a not unreasonable supposition.

From above formula the strength values for different length columns would be as follows :

$H=$ 1	$S=$ 48 000
5	45 410
10	43 250
15	41 320
20	39 420
25	37 650
30	35 810
40	32 200
50	28 690
60	25 380

These values with the exception of the one at 15 diameters show only a discrepancy of from one to five per cent.

The value derived from experiments of such magnitude, as in the foregoing paper published, can hardly be underrated and will be thankfully received by the profession.

Discussion by James E. Howard, C. E.

The elastic limit of the Phœnix columns was established at that load where the apparent decrements of length ceased to be proportional to the increments of load which had caused them.

In the absence of any definition, generally accepted, for the elastic limit, it has been customary with the Watertown Arsenal experiments to supply such data as would show the behavior of the material under test. With some tensile specimens the elastic limit is so sharply defined that there would be no difference of opinion concerning it. In other cases its exact determination might be a matter of doubt. Subsequent tests of Phœnix columns, where the gauging-rods did not extend to the compression platforms of the testing machine, but were attached to the column, showed smaller permanent sets than were found in this first series of experiments. Hence, it was probably correct to attribute a part of the permanent sets found in these experiments to the condition of the ends of the columns and not to the metal itself. The manner of failure of latticed columns referred to one lot only, and as the results were not presented, as I supposed they were to be, remarks concerning them should not have appeared at this time.

Discussion by Thomas C. Clarke, M. A. S. C. E.

It is very gratifying to the writer to know that the publication of the Watertown experiments on Phœnix columns meets with the approbation of so many eminent engineers who have taken part in this discussion.

It is not too much to say that the paper and discussion together form a positive addition to our scientific knowledge.

They have demonstrated that the mathematical theory of columns is correct; and by slightly changing the constants of the usually received formula, as Mr. Bouscaren has done, or by a very simple new formula, as given by Mr. Whittemore, we can interpolate the strength of intermediate lengths and sections of Phœnix colums, within the limits of these experiments.

Some criticism has been made upon what Mr. Howard says in reference to lattice columns. The report from Watertown contained some experiments on lattice columns which have no connection with these Phœnix column experiments, and they were omitted from the manuscript but by an oversight, the sentence objected to was not stricken out at the same time, as it should have been.

It is hoped that others who have in their possession data obtained from experiments on the United States Testing Machine, will send them to this Society for publication, that they may have the great advantage of discussion by experts, as these experiments have had.

It is hoped that a series of experiments will soon be made upon similar Phœnix columns of steel, and that the results will be published and discussed by this Society.

In the paper it was stated that the experiments were made at the cost of Clarke, Reeves & Co.

The correct statement is, that these columns were ordered of the Phœnix Iron Company by the old United States Testing Board at a cost of $775 00, and were paid for by that Board. That there being no money available to test them, the charge for this was paid by Clarke, Reeves & Co., and amounted to the sum of $439 05. This statement is made, not only to correct what might be misunderstood, but to show how desirable it is that certain appliances should be added to the present United States Testing Machine, for the better handling of specimens, etc., so that the cost of testing may be reduced from 45 per cent. of the cost of the material, down to not over 10 per cent. at least.

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TRANSACTIONS.

NOTE.—This Society is not responsible as a body, for the facts and opinions advanced in any of its publibations.

CCXXXIV.

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AVERAGING MACHINE.

By W. S. Auchincloss, Member of the Society.

Read March 1, 1882.

During the past year the averaging machine, illustrated by Plate XXIII of Vol. X, has been remodeled and is now presented in a more compact and useful form, with wider range and greater capacity. In the original design, the representative weights on the platform were exactly balanced by equal weights in the scale pan, on the opposite side of the fulcrum. In the present device, the scale pan has been discarded and the representaitve weights are made to balance themselves over a common fulcrum *a a*. By this means the weights handled are reduced one-half in amount, and the chances of error greatly diminished.

In order to have the weights retained at certain relative distances and at the same time determine the location of their centre of gravity, it is necessary to hold them on a perfectly balanced platform, so that the weight of the platform will not enter the problem. There are many ways in which this factor can be eliminated and the desired result secured. For instance, the divisions between weights can be made part of an endless chain passing over and under the platform, or the platform itself might readily be balanced by a ball and lever ; but, probably, one of the simplest forms is that shown in the accompanying cuts.

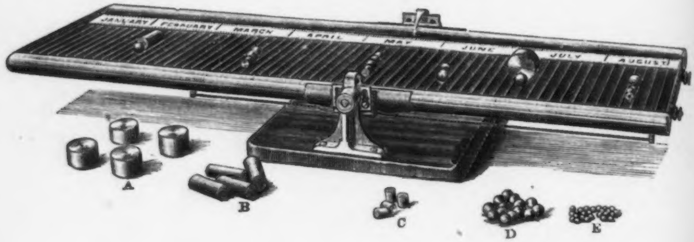

FIG. 1.

The grooved platform rests on a saddle, C, which in turn is suspended on the knife edges a, a. The saddle C carries four pulleys, d, d', d'', d''', on its under side. These pulleys have grooved edges to receive chains or wire ropes, g, g', g''. g'''. The saddle C has also a rib on its under side, to secure the required amount of rigidity. The knife edges are secured to the saddle by screws and slotted plates, so that the position of the centre of gravity of the platform and saddle can be varied at pleasure in a vertical direction, and a proper degree of stability secured. The edges of the platform are formed by two tubes, B, B', slotted on their under sides. These .tubes carry the counterweights F, F'. The counterweights have the ends of the chains g, g', g'', g''', fastened to their extremities at f, f', f'', f''', and the other extremities of the chains are fastened to small screw-bolts at h, h', h'', h'''. The screw-bolts pass through the ends of the platform and the chains are drawn taut by milled nuts, and kept so by lock nuts. The knife edges, or trunnions of the saddle, rest on the standards b, b.

When the machine is first put together, the saddle is hung by its trunnions on the standards and its weight is increased on the lightest side, until the balance is secured. The counterweights are then made of such weight, that they *exactly* equal the combined weight of platform, paper scale, screw-bolts, nuts, and bolt wires, plus the weight of excess of chain (pendant from the platform over that pendant from the extremities of the counterweights). The latter factor will always be a constant quantity. Having secured the above equality, the counterweights are placed in the tubes, the platform in the saddle, and the chains adjusted

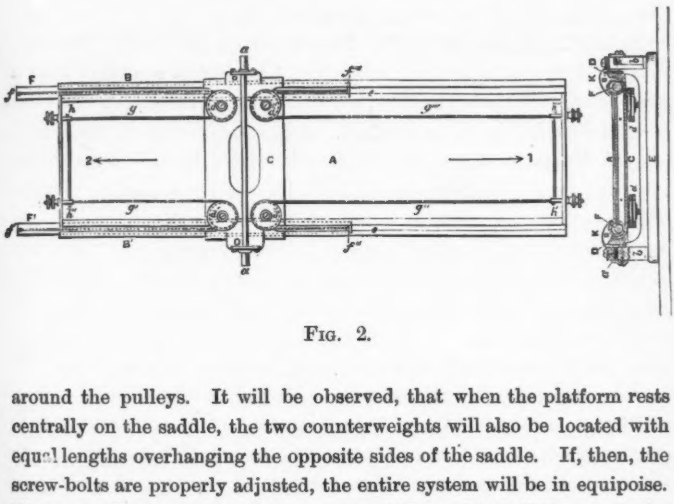

FIG. 2.

around the pulleys. It will be observed, that when the platform rests centrally on the saddle, the two counterweights will also be located with equal lengths overhanging the opposite sides of the saddle. If, then, the screw-bolts are properly adjusted, the entire system will be in equipoise. The next adjustment is to draw out the platform *A* its full distance in the direction of the arrow No. 1. The counterweights will immediately be projected an equal distance in the opposite direction, and as their weight exactly equals the combined weight of the platform and its parts, the equipoise should be as perfect as that found for the mid position of the platform. When both of these adjustments have been carefully made, it will be found that no matter where the platform may be subsequently placed, it will continue in perfect equipoise.

With this result accomplished, we can place on this equilibrated platform any combination of representative weights, and by drawing out the platform to its balancing point, the location of the centre of gravity

of the system will at once be indicated on the scale by the pointers over the trunnions.

The representative weights may for convenience be arranged on the decimal system, thus *E*, *D* and *A*, may represent units, tens, hundreds; while *C* and *B* represent 5 and 50.

The same weights may be used to express multiples, or factors of these quantities, and consequently serve for the solution of a great variety of problems. Each machine is supplied with several paper scales, suitably divided for different purposes.

When the problem is one of time, the scale represents months and days. When one of proportion, the scale has its zero point at the centre of its length, and the subdivisions branch in both directions. When the question is that of location of the centre of gravity of a system from a fixed point, the zero is located at one extremity of the scale; and so on for different objects to be attained. A convenient size of platform is one 29 inches in length by 9 inches in width, having 63 transverse grooves. Such a machine can be made to weigh less than 13 pounds. The range of the platform is not limited by the 63 grooves, for if a weight should happen to fall midway between grooves (*i. e.*, upon the ridge), one-half of the amount can be dropped in one groove and the remaining half in the other, without altering the result. So also it is possible to lodge 3 parts in one groove and 1 part in the other, and secure exactly the same effect as if we had 4 grooves at our command. It is clear, therefore, that 63 grooves can be made to answer the purposes of 126, or of 252 grooves, as the case may require. The final reading of the index pointer can be made with great exactness by means of a finely divided scale, and the exact balance can be indicated by aid of a small spirit level attached to the platform.

With this wide latitude the averaging machine may be used for finding the average date of purchases extending over a period of 8 months. By reversing the paper scale, about the average date as a centre, the credit side of the ledger can be laid off, the paper scale returned to its original position and a new date found, which will express the average of both the debit and credit sides of the account. For purchases covering only one month, the average can be determined to an hour. 100 accounts of this character can readily be solved in one hour's time.

The averaging machine offers the cotton broker the most rapid

means for determining the average price of "futures," whether the same are figured in pence English, or in cents of our own currency.

Thus far we have dwelt only on the commercial aspect of the subject. It also has its direct bearing on the question of average haul.

Omitting the rebate due to "limit of free haul" (which is purely a matter of contract), we have by the principle of moments :

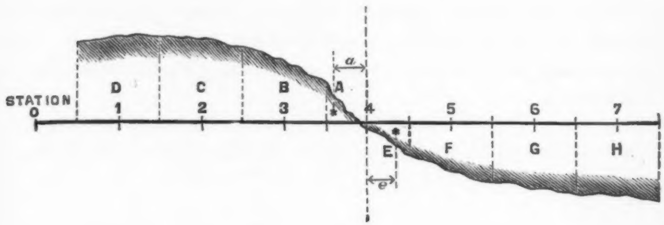

FIG. 3.

With stations 100 feet apart :

$$\text{Average haul} = \frac{(B+2C+3D)\times 100 + Aa,}{A+B+C+D} + \frac{(F+2G+3H)\times 100 + Ee.}{E+F+G+H}$$

Whenever the limit of free haul $=100$ feet, then A, a, E, e are in effect $=$ o, and :

$$\text{Average haul} = \left[\frac{B+2C+3D}{B+C+D} + \frac{F+2G+3H}{F+G+H}\right]\times 100$$

The above formulæ are based on the assumption that each mass B, C, D, &c., has its centre of gravity directly over the respective stations 3, 2, 1, &c.

Whenever a greater degree of accuracy is required the sections can be made of half size, which will give stations 50 feet apart, instead of 100, and the computations can be made accordingly.

By aid of the averaging machine the respective distances between the centres of gravity of the excavation and of the embankment (from the station No. 4) can be severally determined, and their sum will equal the total average haul ; from which the limit of free haul must de deducted.

By this method it is evident that representatives of the amounts in the respective sections of the embankment, need only be placed in proper order on the platform of the averaging machine—a single pull given to secure the equipoise—and the haul of excavation can be instantly read

off from the paper scale. If the same processes are repeated for the embankment, the total average haul will be determined.

The averaging machine can also be used advantageously in the solution of many problems that occur in ship building, location of marine engines, boilers, etc., etc.

A moment's thought will show clearly that the chances of error are greatly reduced, and valuable time redeemed by the use of this machine.

In the wide range of problems relating to averages, probably few will present themselves, which cannot find a ready solution upon an equilibrated platform of greater or less magnitude than the one herewith described.